Modeling assets and liabilities

Covariance estimation

Successful investing requires an understanding of asset and market dynamics. For portfolio management, the covariance matrix is essentially a model of "how the world works" in that it describes both the volatilities of, and the relationships between, investments under consideration. This information is central to the development of efficient portfolios and for risk management exercises. Unfortunately, developing a model of covariance that quantifies how assets might be expected to act in the future is not a simple task. Invesco has expended considerable effort to identify a framework that provides relevant information for portfolio construction and risk assessment. Our focus was on identifying a multi-factor risk model that provided a high degree of flexibility and allowed for the consistent modeling of a broad range of assets.

Multi-factor risk models generally fall into three main categories: 1) macroeconomic models, 2) statistical models, and 3) fundamental models. Macroeconomic models are perhaps the most simple and intuitive in that they use observable economic time series (e.g., GDP, interest rates, inflation, credit spreads, etc.) to explain the returns of, and relationships between, assets. Conversely, statistical factor models are substantially less intuitive because they rely on unobservable statistical factors, generally derived from a factor analysis or principal components analysis, for explanatory purposes. Determining sensitivities, or betas, to either macroeconomic or statistical factors is accomplished through time series regression. This is a key limitation for these types of models in that the statistically sound estimation of exposures requires long return histories. In many instances, sufficiently long historical information is not available and, when it is, it may not reflect the evolution of an asset's characteristics over time.

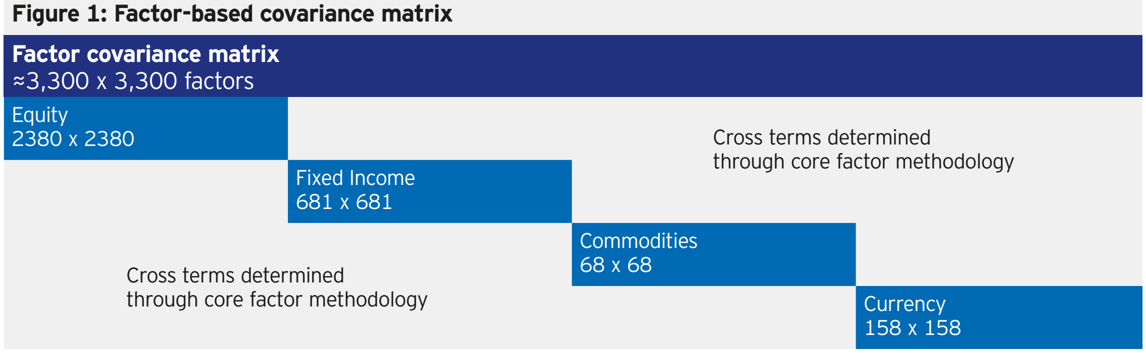
Fundamental factor models do not rely on time series regression for estimating sensitivities but instead use directly observable asset attributes (e.g., industry, price-earnings ratio, price momentum, market capitalization, etc.) to explain returns. These attributes are treated as betas which, when combined with risk indices that correspond to the various attributes identified, allow for the estimation of asset behavior. While all of these risk models have potential benefits, the fundamental risk model provides significant flexibility in practice along with providing for an intuitive understanding of the dependence of an asset's returns on well-defined dependencies. For these reasons, we have selected it to model the risks of the global collection of comprehensive asset classes that are available for use in Invesco Vision. Rather than embarking on the daunting task of developing a proprietary fundamental risk model, Invesco Vision has incorporated BarraOne®, which is one of the most recognized and respected risk models available.

Modeling traditional assets

As briefly discussed above, we have chosen a multi-asset fundamental risk factor model to drive our risk modeling capabilities. The model is a comprehensive one utilizing over 3,000 factors, including factors that span all of the major asset classes across various economies, countries, and industries. One of the key benefits of using a factor model is our ability to distill the key performance drivers of any security into a smaller set of relevant systematic factors. This means that, in most cases, we may need to employ only a small subset of relevant factors for modeling exercises.

Figure 1 presents the overall structure of the factor covariance matrix. The matrix is comprised of four subcomponents: equities, fixed income, commodities, and currencies. Equity factors are further subdivided into industry factors and style factors and fixed income factors are further subdivided into term structure and spread factors.

Figure 1: Factor-based covariance matrix



Source: Invesco, BarraOne.

Producing the covariance matrix relies on looking at factor return series and determining their variances and covariances. Like most risk models, recent data is weighted more heavily than older data. We offer three variants from which to choose as shown in Figure 2 below. Given the strategic nature of most of our modeling efforts, we will most frequently leverage the longer half-life covariance matrix.

Figure 2: Covariance matrix time horizon, half-life, and use cases

	Variance half-life	Covariance half-life	Use case
Short	90 days	2 years	Best suited for tactical positioning
Medium	1 year	3 years	
Long	8 years	8 years	Best suited for strategic asset allocation

Source: Invesco, BarraOne

With a covariance matrix defined, any security or fund can then be modeled through its factor exposures and specific (or idiosyncratic) risk estimates. Figure 3 shows this information for three example securities: a US equity, a EUR denominated bond, and a commodity futures contract. For the US equity example, we can see that the US software firm stock has 100% exposure to the US Software industry factor and additional exposures to a variety of equity style factors which are measured in terms of z-scores. The risk of this stock can then be modeled based on its factor exposures and the underlying factor covariance matrix. Modeling the risk characteristics of portfolios comprised of multiple assets employs the same information.

Figure 3: Sample equity, bond, and commodity future factor exposures (as of 8/31/2018)

Equity: US Software firm		Bond: European financial firm $4^{1/8}$ 1/14/20		Future: Crude Oil Dec 18	
Equity industry		**Term Structure**		**Commodity**	
US Software	100%	EUR Shift	1.34	Commodity crude oil shift	0.96
		EUR Twist	-2.23	Commodity crude oil twist	0.64
		EUR Butterfly	1.62	Commodity crude oil butterfly	0.17
Equity style		**Spread**			
US beta	1.26	EUR Swap shift	1.34		
US Non-linear beta	-1.12	EUR Swap twist	-2.63		
US Residual volatility	-0.74	EUR Swap Butterfly	0.23		
US Book-to-price	-0.63	EUR Financials A	1.33		
US Earnings yield	-0.35				
US Dividend yield	-0.14				
US Momentum	0.92				
US Leverage	-0.26				
US Size	1.03				
US Non-linear size	-0.53				
US Liquidity	-0.94				
US Growth	-0.20				
Currency		**Currency**			
USD	100%	EUR	100%		
Specific Risk	10.1%	**Specific Risk**	0.2%	**Specific Risk**	1.4%

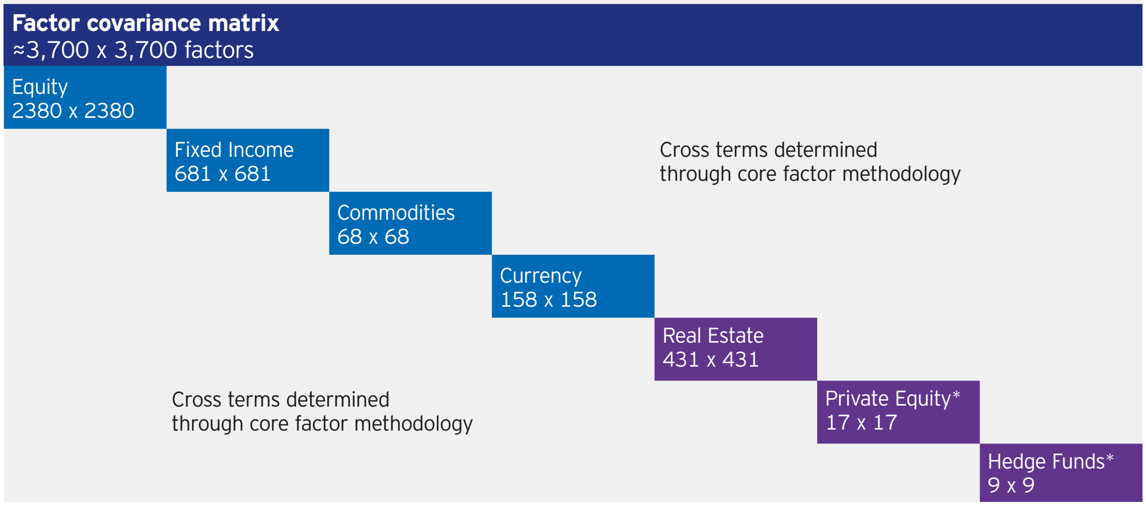
Source: Invesco, BarraOne.

- **Holdings-Based Analysis**
  When the security level holdings of a portfolio are available, we will rely on holdings-based analysis. In this approach, every asset is individually modeled and translated into a set of factor exposures and specific risk estimates, just like the example above. By aggregating the underlying asset exposures, we are then able to effectively model the overall portfolio behavior.
- **Returns-Based Analysis**
  In cases where we do not have information about the underlying portfolio constituents and cannot employ holdings-based analysis, we will proceed with returns-based analysis. In this situation, we rely on the fund's historical returns as well as some high-level fund characteristics, such as asset class, style, and region. We then use a returns-based model to estimate the fund's factor exposures. To do this, a finite set of stylized factor portfolios are employed. The use of stylized portfolios helps address the mismatch between the limited number of historical return data points and the large number of factors in the model. Invesco Vision's model already covers thousands of assets through both holdings- and returns-based analyses. Users can also model a fund on an ad hoc basis using its returns and relevant characteristics.

Modeling alternative assets

Alternative investments can exhibit unique characteristics that can prove beneficial to achieving desired portfolio outcomes. Given their illiquid nature, modeling alternatives presents additional challenges. Generally, de-smoothing techniques that blend private and public factors to achieve more representative risk characteristics must be applied. Figure 4 presents the structure of the factor covariance matrix with alternative factors included. Detail on specific approaches used for modeling various alternative assets follows.

Figure 4: Factor-based covariance matrix including alternative factors



* Private equity and hedge fund assets get exposure to both traditional asset factors as well as the indicated private factors which are uncorrelated to any other factors.
Source: Invesco, BarraOne.

- **Real Estate**
  For private real estate, we model each property based on its property type, subtype, and region. In total, there are 431 real estate factors as shown in Figure 5. Bayesian de-smoothing techniques that include both public and private data are employed to address issues with infrequent, lagged, and non-subjective valuations. When estimating factor exposures, adjustments should consider leverage as indicated by the loan-to-value (LTV) ratio. Furthermore, additional empirical adjustments are made to account for the different behavior of value-add and opportunistic type holdings relative to core holdings. Finally, a specific risk model is also used to capture the idiosyncratic nature of individual holdings.
- **Private Equity and Debt**
  Similar to the real estate model, de-smoothing techniques are employed. However, in this case, assets are assigned exposure to both public and private factors. There is also flexibility to define the public factors to which an asset may be exposed. This allows us, for example, to model a US Technology Buyout fund differently than a European Consumer Cyclical Buyout fund. There are a total of 17 private factors that are delineated by fund type and region as shown in Figure 6.

Figure 5: Private real estate factors
Factor matrix shows number of unique regional factors by property type

Country	Office	Residential	Retail	Industrial	Hotel	Other	Income return	Sub-type	City	Farmland	Agriculture
Australia	5	1	1	3	1	1	1	–	–	–	–
Austria	3	1	1	1	1	1	1	–	–	–	–
Belgium	4	1	1	1	1	1	1	–	–	–	–
Canada	5	1	5	2	1	1	1	–	–	–	–
China	1	1	1	1	1	1	1	–	–	–	–
Czech Republic	1	1	1	1	1	1	1	–	–	–	–
Denmark	8	3	1	1	1	1	1	–	–	–	–
France	6	4	1	1	1	1	1	–	–	–	–
Germany	8	1	2	1	1	1	1	–	–	–	–
Hong Kong	1	1	1	1	1	1	1	–	–	–	–
Hungary	1	1	1	1	1	1	1	–	–	–	–
Indonesia	1	1	1	1	1	1	1	–	–	–	–
Ireland	3	1	5	3	1	1	1	–	–	–	–
Italy	5	1	1	1	1	1	1	–	–	–	–
Japan	7	4	3	1	1	1	1	–	–	–	–
Korea	5	2	2	2	1	1	1	–	–	–	–
Malaysia	1	1	1	1	1	1	1	–	–	–	–
Netherlands	4	4	4	4	1	1	1	–	–	–	–
New Zealand	2	2	2	2	1	1	1	–	–	–	–
Norway	8	1	1	1	1	1	1	–	–	–	–
Poland	1	1	1	1	1	1	1	–	–	–	–
Portugal	6	1	1	2	1	1	1	–	–	–	–
Singapore	1	1	1	1	1	1	1	–	–	–	–
South Africa	3	1	1	1	1	1	1	–	–	–	–
Spain	5	1	1	1	1	1	1	–	–	–	–
Sweden	6	4	1	1	1	1	1	–	–	–	–
Switzerland	6	6	2	1	1	1	1	–	–	–	–
Taiwan	1	1	1	1	1	1	1	–	–	–	–
Thailand	1	1	1	1	1	1	1	–	–	–	–
United Kingdom	5	1	3	3	1	1	1	16	–	1	1
United States	4	4	4	4	1	1	1	15	17	11	4

Source: Invesco, BarraOne.

Figure 6: Private equity factors

Fund type	US	Europe	Asia
Large buyout	✓	✓	✓
Small buyout	✓	✓	✓
Early stage venture	✓	✓	✓
Late stage venture	✓	✓	✓
Distressed	✓	✓	✓
Mezzanine	✓	✓	–

Source: Invesco, BarraOne.

■ Hedge Funds

Hedge funds are possibly the hardest of investment types to model. This should be expected as they are very idiosyncratic by nature. Hedge funds are always modeled using returns-based analysis. As a first step, it is necessary to define the hedge fund strategy type. Based on this, a regression is performed against a set of relevant public factors. For a subset of strategies, a hedge fund specific factor is also included in the regression. For example, a merger arbitrage hedge fund will be regressed against the relevant region MSCI IMI index factors, the size factor, as well as a hedge fund "event driven" factor. Figure 7 presents the available hedge fund strategies and corresponding hedge fund and public factors used for modeling exercises.

Figure 7: Hedge fund factors

Hedge Fund Strategy	Hedge Fund Factor	Public Factor (exposure driven by user input, historical fund returns and region)
Long/Short Equity	–	MSCI IMI; Style factors; Currency factors
Dedicated Short Bias Equity	–	MSCI IMI; Style factors; Currency factors
Long/Short Credit	–	High Yield; Term structure factors; EM credit spread factors
Equity Market Neutral	Equity Market Neutral	MSCI IMI; Style factors
Merger Arbitrage	Event Driven	MSCI IMI; Size factor
Event Driven Multi-Strategy	Event Driven	MSCI IMI; Size factor
Distressed Securities	Event Driven	MSCI IMI; High Yield; Size factor
Convertible Arbitrage	Convertible Arbitrage	MSCI IMI; High Yield
Fixed Income Arbitrage	Fixed Income Arbitrage	High Yield; Term structure factors; EM Credit Spread factors
Global Macro	Global Macro	MSCI IMI; Currency factors
Managed Futures	Managed Futures	MSCI IMI; Dollar Index; Currency factors
Fund of Hedge Funds	Event Driven, Global Macro, Managed Futures	MSI IMI; Style factors; High yield; EM credit spread factors

Source: Invesco, BarraOne.

- **Custom alternative assets:**
  While the approaches above can be used to model many alternative assets, there are cases that require more customized modeling. For example, how do we model private infrastructure? How about private commercial real estate debt? While it can be difficult to model these private assets without introducing additional factors, we make the best effort to represent them using the available factors. Given the lack of transparency, limited data, and possible lack of perfect mark-to-market pricing, we choose to keep these modeling exercises as simple as possible.

  For example, for fixed income like alternative assets we may consider rate exposures, credit exposures, and other private factor exposures. Rate exposure is dictated by the nature of the debt. Floating rate debt receives no rate exposure while fixed rate debt includes exposure to the relevant curve shift and twist factors based on the asset's maturity. For credit and private factor exposure, we qualitatively decide which factors are the most relevant. Once we select the factors, we set exposures such that the modeled risk is aligned with our best estimate of the risk of the asset. Finally, the distribution of risk between the public credit and private factors can take on three possible values: 25/75, 50/50, or 75/25. The split is selected through an evaluation of historical correlations with the overall equity and debt markets. In Figure 8 we present examples of various alternative assets and how they might be modeled.

Figure 8: Sample alternative asset factor exposures

Core office building Boston (LTV = 25%)	
Real Estate	
US Office east	133%
US Income return	133%
US Boston	133%
Currency	
USD	100%
Specific risk	8.4%

Private equity large buyout fund	
Equity Industry	
US Aerospace & Defense	3.1%
US Banks	8.5%
US Biotech life sciences	4.7%
US Computer electronics	5.2%
US Diversified financials	5.9%
US Health care equipment and technology	3.5%
...	
Total	**120%**
Private	
US PE large buyout	100%
Currency	
USD	100%
Specific risk	2.1%

Event driven hedge fund	
Equity Industry	
US Aerospace & Defense	0.8%
US Banks	1.3%
US Biotech life sciences	1.0%
US Computer electronics	2.2%
US Diversified financials	2.8%
US Health care equipment and technology	1.5%
...	
Total	**30.5%**
Equity Style	
US Size	-0.02%
Hedge fund	
Pure event driven	73%
Currency	
USD	100%
Specific risk	1.3%

Infrastructure fixed debt	
Term Structure	
US Shift	10.0
US Twist	6.8
Spread	
US Swap shift	10.0
US Utilities BBB	5.1
Private	
US PE mezzanine	35%
Currency	
USD	100%
Specific risk	-

Source: Invesco, BarraOne.

Modeling liabilities
Defeasing a set of liabilities is a common objective for asset managers. Defined benefit plans are managed to support pension liabilities. Insurance companies are managed to meet expected claims. Even retail investors invest with the objective of paying for their future needs. In all these cases, effectively modeling the underlying liabilities is critical.

In order to model a liability stream there are two key ingredients:

1. A cash flow schedule
2. A discount curve

While cash flow schedules are rarely known with certainty, we assume that they are best modeled by the asset owner and as result we treat them as deterministic. For the discount curve we rely on an array of built-in curves based on specific liability characteristics. For nominal cash flows we use nominal yield curves. For inflation adjusted cash flows we use real yield curves. Some curves are based on sovereign rates, others on swap rates, and yet others are based on corporate rates. Figure 9 lists examples of the discount curves used for different types of liabilities:

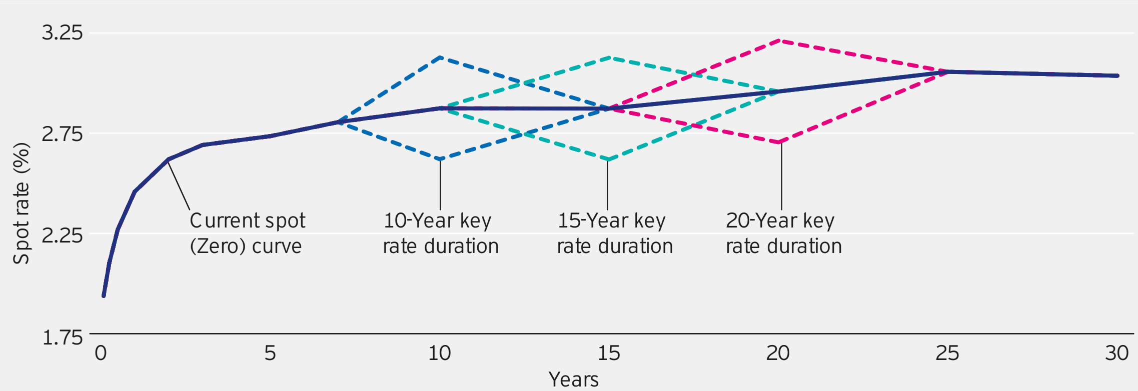
Figure 9: Liability discount curve variants

Liability type	Example curve	Sovereign nominal	Sovereign real	Swap spread	Credit
US Corporate Pension	Citi AA	✓		✓	✓
UK Inflation-linked liability	Indexed Gilt Curve		✓		
European Insurer	EIOPA EUR	✓		✓	

Source: Invesco, Citigroup, AA Pension discount curve, Bloomberg, EIOPA Solvency II discount curve.

The key aspect of modeling any liability stream is understanding how its present value will react to changes in market conditions. More specifically, we need to translate the liability stream into a set of factor exposures just like those for assets. To do this, we model the liability as if it were a bond with unequal interest payments. Typically, the largest risk entailed in a set of liabilities is interest rate risk – i.e., the impact of interest rate movements on its present value. To better understand this risk, we compute the impact of various interest rate shocks. The shocks we choose to examine are identical to the ones we use when we evaluate the interest rate risk of our bond assets. More specifically, we compute key rate durations at a set of pre-specified key rate points. This is a standard approach that entails re-valuing the liabilities based on "hut like" linear dislocations of the interest rate curve as shown below:

Figure 10: Key-rate duration calculation methodology



Source: Invesco, BarraOne.

The exercise requires attention to detail. For example, are we looking for spot key rate durations or par key rate durations? Or does the curve entail a credit component or is it entirely based on a sovereign or swap curve? Seemingly small differences can meaningfully impact the resulting analytics and could lead to unnecessary model risk.

For curves that entail a credit component, such as the FTSE pension discount curve, there is the additional challenge of credits migrating into and out of the curve. To model these types of curves there are several options of varying sophistication that can be employed. Given that we are generally managing multi-asset portfolios, where even small exposures to growth assets may dominate risk, we follow a straightforward approach. Specifically, we model the credit portion of the discount curve using a generic AA corporate spread factor and set the exposure level to be equal to the overall duration of the liability stream.[1]

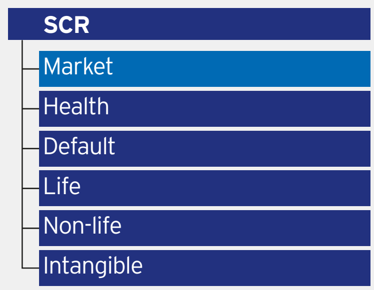
Regulatory risk models

Invesco Vision also allows insurance entities operating in either the Solvency II framework or the NAIC framework to develop capital-efficient investment portfolios. Depending on the regulatory jurisdiction, insurers must set aside capital as a cushion to protect against adverse movements in their asset portfolios. Each of these frameworks use their own formulaic methodologies for computing the capital charges that will be applied to various asset allocation schemes.

Solvency II

For Solvency II we focus on the Market Risk component of the Solvency Capital Requirement (SCR) calculation as shown in Figure 11. This is comprised of six sub-components: Interest rate risk, spread risk, equity risk, property risk, concentration risk and currency risk:

Figure 11: Solvency II Solvency Capital Requirement (SCR) modules

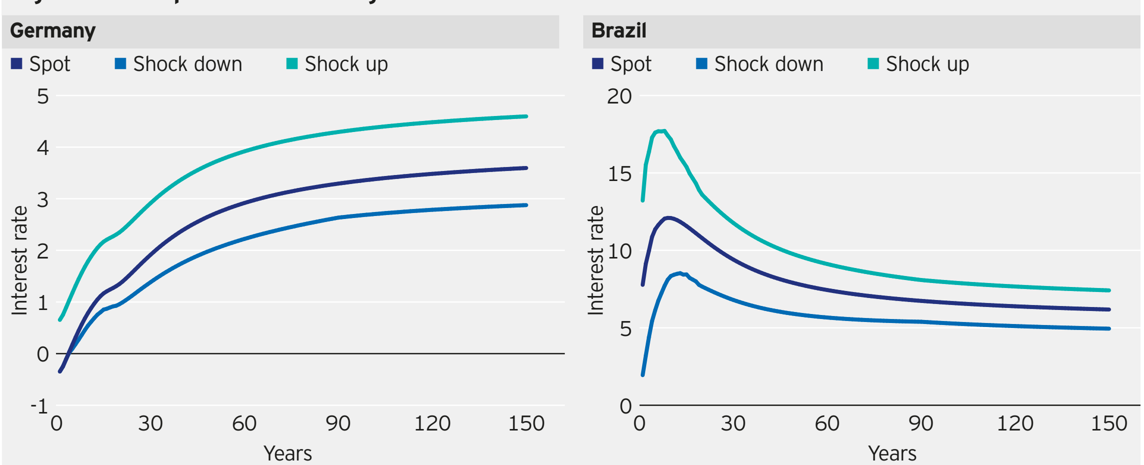


Source: Invesco, Solvency II Directive.

Interest rate risk:

All interest rate sensitive assets and liabilities are exposed to an upward and downward interest rate shock. The shocks are prescribed by the European Insurance and Occupational Pensions Authority (EIOPA) for various sovereign curves. Two examples are shown in Figure 12.

Figure 12: Sample EIOPA sovereign rate curves with curve shocks



Source: Invesco, EIOPA. Data as of Aug. 31, 2018.

1 This approach is used only to estimate the risks of the liabilities. The present value is always computed based on the exact discount curve.

A pricing engine is used to compute the impact of both the upward and downward shock to the assets and the liabilities. This way, each asset and the liabilities have two possible interest rate charges – an upward shock charge and a downward shock charge. Figure 13 shows the resulting charges of various duration liabilities in different regions.

Figure 13: Example SCR charges for various duration liabilities (%)

	USD			EUR			BRL		
	Duration	SCR_{IRup}	SCR_{IRdown}	Duration	SCR_{IRup}	SCR_{IRdown}	Duration	SCR_{IRup}	SCR_{IRdown}
SCG 4	4.67	6.06	5.44	5.03	4.83	0.96	3.27	16.84	18.93
SCG 10	12.67	12.62	11.87	13.34	12.21	5.28	7.64	27.79	37.72
SCG 16	20.67	18.77	18.44	20.85	18.43	10.83	12.93	35.30	58.25

Source: Invesco, EIOPA, BarraOne, Russell standard cash flow generator.

Spread risk:
Asset spread SCR charges are computed based on the issuer type, the asset rating, and the asset spread duration. There are three types of assets with spread risk charges: bonds and loans, securitized assets, and derivatives.

- **Bonds and Loans**
  Bonds and loans are further grouped into three categories each with its own treatment: Corporate bonds and loans, infrastructure bonds, and government bonds.
  - **Corporate bonds and loan**
    Corporate bonds and loans entail all bonds that do not fall in the other spread categories. SCR charges for these bonds depend on their rating and spread duration. Figure 14 provides sample SCR charges for specific rating/duration combinations.

Figure 14: Sample corporate bond and loan SCR charges (%)

Spread Duration	AAA	AA	A	BBB	BB	B or less	NR
1-Year	0.9	1.1	1.4	2.5	4.5	7.5	3.0
5-Year	4.5	5.5	7.0	12.5	22.5	37.5	15.0
10-Year	7.0	8.5	10.5	20.0	35.0	58.5	23.5
20-Year	12.0	13.5	15.5	30.0	46.5	63.5	35.5

Source: Invesco, Solvency II Directive.

  - **Infrastructure Bonds**
    Infrastructure debt is treated more favorably than corporate debt. To qualify for this treatment, the infrastructure project must be located in the European Economic Area (EEA) or Organization for Economic Co-operation and Development (OECD) regions. In the case that the debt is not rated but is senior to all other claims, it will be treated as a BBB rated issue. Any infrastructure projects that are rated below BBB do not qualify. The SCR charge structure is like that of corporates as shown in Figure 15.

Figure 15: Sample infrastructure bond SCR charges (%)

Spread Duration	AAA	AA	A	BBB	BB	B or less	NR
1-Year	0.64	0.78	1.00	1.67	-	-	-
5-Year	3.20	3.90	5.00	8.35	-	-	-
10-Year	5.00	6.05	7.50	13.35	-	-	-
20-Year	8.60	9.65	11.10	20.05	-	-	-

Source: Invesco, Solvency II Directive.

  - **Government Bonds**
    Bonds issued by the European Central Bank or central governments and banks of member states denominated in local currency are exempt from SCR charges. Bonds issued by other central banks or countries denominated in their local currency do receive SCR charges. Figure 16 presents the SCR charge structure for government bonds.

Figure 16: Sample government bond SCR charges (%)								
	European Sovereign Bonds	Non-European Sovereign Bonds						
Spread Duration	Any Rating	AAA	AA	A	BBB	BB	B or less	NR
1-Year	0.0	0.0	0.0	1.1	1.4	2.5	4.5	-
5-Year	0.0	0.0	0.0	5.5	7.0	12.5	22.5	-
10-Year	0.0	0.0	0.0	8.5	10.5	20.0	35.0	-
20-Year	0.0	0.0	0.0	13.4	15.5	30.0	46.5	-

Source: Invesco, Solvency II Directive.

- **Securitized assets**
  Securitized assets are treated punitively in Solvency II. There are three types of securitized assets distinguished generally as Type I, Type II, or Re-securitizations. For an asset to qualify for Type I securitization it must be listed in an EEA or OECD country and needs to also be the most senior tranche. Furthermore, the underlying loans need to be homogeneous and not include re-securitization. Any securitized asset that is not Type I or re-securitized is considered Type II. The SCR charges are based on the type of securitization, the rating, and the spread duration of the underlying asset. Figure 17 presents SCR charges for hypothetical five-year duration securitized assets.

Figure 17: Sample SCR charges for securitized bonds with a five-year spread duration (%)							
Spread Duration	AAA	AA	A	BBB	BB	B or less	NR
Type I	10.5	15	15	15	-	-	-
Type II	62.5	67	83	98.5	100	100	100
Re-Sec	100	100	100	100	100	100	100

Source: Invesco, Solvency II Directive.

- **Derivatives**
  The treatment of credit derivatives depends on whether they are used for hedging or opportunistic bets. Asymmetric absolute and relative shocks should be applied to compute resulting charges.

The total SCR spread charges are computed as the sum of the underlying charges as below:

$$SCR_{SPREAD} = SCR_{bonds} + SCR_{sec} + SCR_{der}$$

Equity risk[2]:
There are three types of equity charges as indicated in the following table:

Figure 18: Equity SCR charges (%)		
Spread Duration	Description	Charge
Type I	Listed equities in EEA and OECD countries	39
Type II	Listed equities not in EEA or OECD countries and unlisted equities	49
Infrastructure	Qualifying infrastructure equities	30

Source: Invesco, Solvency II Directive.

2 Assets not covered by other modules are also treated as equity Type 2 assets. This includes commodities, alternative investments, and any other assets for which look-through is not possible.

The total equity SCR is then calculated as follows:

$$SCR_{EQUITY} = \left[SCR_{E1}^2 + (SCR_{E2} + SCR_{Einf})^2 + 2 \times 0.75 \times SCR_{E1} \times (SCR_{E2} + SCR_{Einf})\right]^{\frac{1}{2}}$$

Property risk:
All direct real estate holdings are charged at a 25% rate. In the case that any leverage is employed it needs to be accounted for by accordingly adjusting the charge.

Concentration risk:
Concentration risk is related to having too much exposure in a single entity. This calculation is highly non-linear and must be calculated separately.

Currency risk:
All assets that are denominated in a currency that is different from the insurers base currency will be charged at a 25% rate.

Once all the subcomponents of the SCR charges are computed, the total market risk SCR charge can finally be computed as follows:

$$SCR_{MARKET} = \left[\left(\overrightarrow{SCR_{MR}}\right)^{*} \cdot \Sigma^{\pm}_{SCR} \cdot \overrightarrow{SCR_{MR}}\right]^{\frac{1}{2}}$$

In the above formula, SCR_{MR} refers to the vector of the six SCR charges outlined above and $\Sigma^{\pm}_{SCR}$ refers to the correlation matrix which can take on two possible values as shown in Figure 19.

Figure 19: SCR Upward and downward correlation matrices

	Upward Shock						Downward Shock					
	SCR_{IR}	**SCR_{SP}**	**SCR_{CO}**	**SCR_{FX}**	**SCR_{EQ}**	**SCR_{PR}**	**SCR_{IR}**	**SCR_{SP}**	**SCR_{CO}**	**SCR_{FX}**	**SCR_{EQ}**	**SCR_{PR}**
SCR_{IR}	**1**	0	0	0.25	0	0	**1**	0.5	0	0.25	0.5	0.5
SCR_{SP}	0	**1**	0	0.25	0.75	0.5	0.5	**1**	0	0.25	0.75	0.5
SCR_{CO}	0	0	**1**	0	0	0	0	0	**1**	0	0	0
SCR_{FX}	0.25	0.25	0	**1**	0.25	0.25	0.25	0.25	0	**1**	0.25	0.25
SCR_{EQ}	0	0.75	0	0.25	**1**	0.75	0.5	0.75	0	0.25	**1**	0.75
SCR_{PR}	0	0.5	0	0.25	0.75	**1**	0.5	0.5	0	0.25	0.75	**1**

Where:

SCR_{IR} = Interest rate risk charge
SCR_{SP} = Spread risk charge
SCR_{CO} = Concentration risk charge
SCR_{FX} = Currency risk charge
SCR_{EQ} = Equity risk charge
SCR_{PR} = Property risk charge

Source: Invesco, Solvency II Directive.

An SCR_{Market} value is computed based on each of the above correlation matrices using the corresponding SCR_{ir_up} and SCR_{ir_down} asset and liability charges. The final value of SCR_{Market} is the worst of the two values.

National Association of Insurance Commissioners (NAIC) Risk-Based Capital (RBC)

Similar to the Solvency II framework, the RBC framework in the US is also comprised of multiple modules. In this case, the modules vary based on the insurance entity type as shown in Figure 20. Invesco Vision addresses only the asset risk modules that focus on investment risks associated with fixed income and equity.

Figure 20: NAIC RBC insurance company modules

RBC - Life
- C_0: Affiliate risk
- C_{1cs}: Asset risk Common stock
- C_{10}: Asset risk - All other
- C_2: Insurance risk
- C_{3a}: Interest rate risk
- C_{3b}: Health credit risk
- C_{3c}: Market risk
- C_{4a}: Business risk
- C_{4b}: Business Risk - Health

RBC - Property
- R_0: Affiliate risk
- R_1: Asset risk - Fixed income
- R_2: Asset risk - Equity
- R_3: Credit risk
- R_4: Underwriting risk
- R_5: Underwriting risk
- R_{cat}: Catastrophe risk

RBC - Health
- H_0: Affiliate risk
- H_1: Asset risk
- H_2: Underwriting risk
- H_3: Credit risk
- H_4: Business risk

Source: Invesco, NAIC.

Fixed income credit risk

This module captures credit related charges. The key driver of the charges is the underlying NAIC designation of the fixed income assets. The charges also vary based on the type of insurance entity as shown in Figure 21.

Figure 21: Fixed income credit risk RBC charges (%)

NAIC Designation	Life (pre-tax)	Life (post-tax)	Property & Casualty	Health
NAIC 1	0.40	0.30	0.30	0.30
NAIC 2	1.30	0.96	1.00	1.00
NAIC 3	4.60	3.39	2.00	2.00
NAIC 4	10.00	7.38	4.50	4.50
NAIC 5	23.00	16.96	10.00	10.00
NAIC 6	30.00	19.50	30.00	30.00

Source: Invesco, NAIC.

For most fixed income securities, NAIC designations follow a mapping of Nationally Recognized Statistical Ratings Organizations (NRSROs) ratings:

NAIC 1 = A-rated and above
NAIC 2 = BBB-rated
NAIC 3 = BB-rated
NAIC 4 = B-rated
NAIC 5 = CCC-rated
NAIC 6 = Below CCC

Non-fixed income asset risk

This module includes all non-fixed income assets, such as equity and real estate. Figure 22 outlines these charges for the four types of insurance entities.

Figure 22: Non-fixed income RBC charges (%)

NAIC Classification	Life (pre-tax)	Life (post-tax)	Property & Casualty	Health
Equity	30	19.50	15	15
Real Estate	15	9.75	10	10

Source: Invesco, NAIC.

Once all the RBC module charges are computed, the total RBC charge can be determined formulaically. Below we indicate the equations for various insurance entities where risks are indicated by the risk modules for each entity type:

$$RBC_{LIFE} = C_0 + C_{4a} + \sqrt{(C_{1o}+C_{3a})^2 + (C_{1cs}+C_{3c})^2 + C_2^2 + C_{3b}^2 + C_{4b}^2}$$

$$RBC_{P\&C} = R_0 + \sqrt{R_1^2 + R_2^2 + R_3^2 + R_4^2 + R_5^2 + R_{cat}^2}$$

$$RBC_{HEALTH} = H_0 + \sqrt{H_1^2 + H_2^2 + H_3^2 + H_4^2}$$

Based on the above equations we introduce the correlation matrices for the two risk categories covered by Invesco Vision in Figure 23.

Figure 23: RBC correlation matrices for various insurance entities

	Life		Property & Casualty		Health	
	Fixed income	Non fixed income	Fixed income	Non fixed income	Fixed income	Non fixed income
Fixed income	**1**	0	**1**	0	**1**	1
Non fixed income	0	**1**	0	**1**	1	**1**

Source: Invesco, NAIC.

It is worth noting here a key difference between Solvency II in Europe and RBC in the United States. Whereas Solvency II requires look-through to the underlying holdings of mutual funds, RBC does not. This means an insurer in Europe has the same effective capital charge whether they invest in bonds directly or via a mutual fund; but for US insurers, unrated bond funds will be treated as equity investments, resulting in a much higher capital charge compared to holding the bonds directly.

Asset level covariance matrix

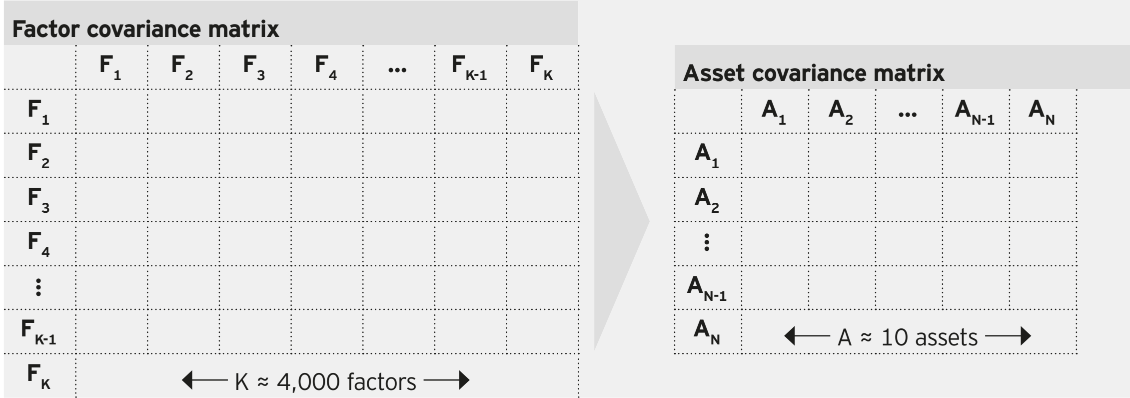
Given either the economic, Solvency II, or NAIC risk factor exposures and their associated risk factor covariance or correlation matrices, we ultimately wish to construct an asset level covariance matrix. We wish to translate the potentially large number of risk factors or capital risk charges into estimates of asset level covariances. For N assets, this results in a compact N by N risk matrix that can be used for portfolio construction and risk estimation exercises.

For the economic risk model, once we have the vector of factor exposures for each asset, we group these vectors into a matrix of factor exposures. We then multiply the factor exposures with the factor covariance matrix to produce the asset level covariance matrix. Namely, given the K by 1 factor exposure vectors β_j *for* $j = 1, 2, \ldots, N$, and the K by K factor covariance matrix Σ_f, we compute the economic asset level covariance matrix Σ_A as follows:

$$\Sigma_A = \left(\beta_1\ \beta_2 \cdots \beta_N\right)^T \Sigma_f \left(\beta_1\ \beta_2 \cdots \beta_N\right) + \Lambda$$

where $\Lambda = diag\left(\sigma_1^2, \sigma_2^2, \cdots, \sigma_N^2\right)$ is a diagonal matrix of idiosyncratic or specific risks. In Figure 24 we illustrate the dimension reduction of the nearly 4,000 by 4,000 factor covariance matrix to an asset covariance matrix.

Figure 24: Converting factor covariances to asset covariances



Source: Invesco, BarraOne.

For the Solvency II risk model, we group the solvency risk factors in a similar fashion and compute the matrix product where we now use the 6 by 1 Solvency risk vectors βj for *j =1, 2, …, N* and the 6 by 6 risk up/down correlation matrices $\Sigma_{SCR}^{\pm}$ (corresponding to prescribed up and down interest rate shocks) and define the asset level covariance Solvency II risk covariance matrix as follows:

$$\Sigma_A = (\beta_1 \ \ \beta_2 \cdots \beta_N)^T \Sigma_{SCR}^{\pm} (\beta_1 \ \ \beta_2 \cdots \beta_N)$$

Finally, for the NAIC risk model we construct the asset level risk covariance matrix through a similar process to the one followed for Solvency II. However, for NAIC there are two risk categories and there is only one correlation matrix Σ_{NAIC}, hence the asset level covariance is formed through the following matrix product:

$$\Sigma_A = (\beta_1 \ \beta_2)^T \Sigma_{NAIC} (\beta_1 \ \beta_2)$$

Estimating expected returns

Having established a process to estimate the risk of various assets, we also need to be able to estimate their returns. To do this we rely on Invesco's capital market assumptions (CMAs) that cover a broad number of asset classes across multiple regions of the global economy. However, despite the extensive coverage, there will be cases where our asset blocks do not perfectly align with our CMA asset coverage. As a result, and in order for us to systematically assign returns to any asset block, we have developed a framework that leverages the underlying factor exposures of our CMA and non-CMA assets.

The algorithm aims to create a replicating (minimum tracking error) portfolio of CMA assets for any asset we want to further evaluate. This portfolio is created by leveraging the factor exposures and the relative optimization framework that will be discussed later. We then assume that the replicating portfolio, comprised entirely of CMA assets, should provide a reasonable estimate of the return of the asset in question. The return of the asset is estimated as follows:

$$R_{Asset} = w_1 R_{CMA\text{-}1} + w_2 R_{CMA\text{-}2} + \cdots + w_N R_{CMA\text{-}N} + e$$

where w_j are the CMA asset weights that sum to 100%, R_{CMAj} is the *j*'th forecasted CMA asset's return, and *e* is the residual error.

In most cases, the algorithm is very effective in identifying a CMA asset portfolio that closely tracks the asset being evaluated. However, in some situations, where the asset lies in a space that is not covered by our CMAs, replication can be more difficult. Invesco Vision will alert the user to instances when assets can not be tracked well. In such cases, a thorough, manual review of the estimate is recommended where a return override can be input into the system.

Depending on the specific problem, a user can choose to employ 10-year horizon CMAs or five-year horizon CMAs. Also, for fixed income, it is possible to use the yield as an estimate of return, entirely ignoring the CMAs. Finally, a user may wish to utilize their own CMAs, in which case they would need to input them directly into the system. Figure 25 provides an example of various expected return possibilities.

Figure 25: Example of expected return selection

Asset	Invesco CMA 10-year (%)	Invesco CMA 5-year (%)	Yield / CMA 10-year (%)	Yield / CMA 5-year (%)	User (%)
US large cap equity	6.5	5.7	6.5	5.7	
US small cap equity	7.9	7.9	7.9	7.9	
Europe ex UK equity	7.3	7.4	7.3	7.4	
UK equity	8.4	8.2	8.4	8.2	
APAC ex Japan equity	9.5	10.9	9.5	10.9	
Japan equity	5.6	5.6	5.6	5.6	
Emerging market equity	9.5	10.4	9.5	10.4	

Source: Invesco.

Arithmetic versus geometric returns

In practice, asset returns are most commonly expressed in geometric terms. This is because the investors are most often concerned with either the rate at which an investment grew in the past or the rate it might be expected to grow in the future (or over the long term). The geometric mean return is the average rate of return per period when returns are compounded over multiple periods. Consider a time series of returns r_t, for $t = 1, 2, \cdots, T$ periods, and some initial investment amount W_0. The value of the investment at time T is $W_T = W_0 \times (1 + r_1) \times (1 + r_2) \cdots \times (1 + r_T)$. The geometric return μ_g, or geometric mean, of such a time series is then:

$$\mu_g = \left(\prod_{t=1}^{T} (1 + r_t) \right)^{1/T} - 1$$

The geometric mean return is of interest to investors because it neatly expresses the periodic growth rate of a time series, i.e., $W_T = W_o(1+\mu_g)^T$. This is of practical importance in terms of understanding the desirability of one investment over another. However, the geometric mean says nothing about risk, or rather, the variability of the returns an investor might actually receive from one period to the next. In fact, two assets can have the same geometric mean but exhibit substantially different variability of returns. To consider risk we must understand the expected value of the return we might receive in any period along with the variability around that expected value. This is where expressing returns in arithmetic terms is useful for investors.

The arithmetic mean μ_a is just the simple average of the periodic returns produced by an asset over a specified investment horizon and is calculated as:

$$\mu_a = \frac{1}{T} \sum_{t=1}^{T} r_t$$

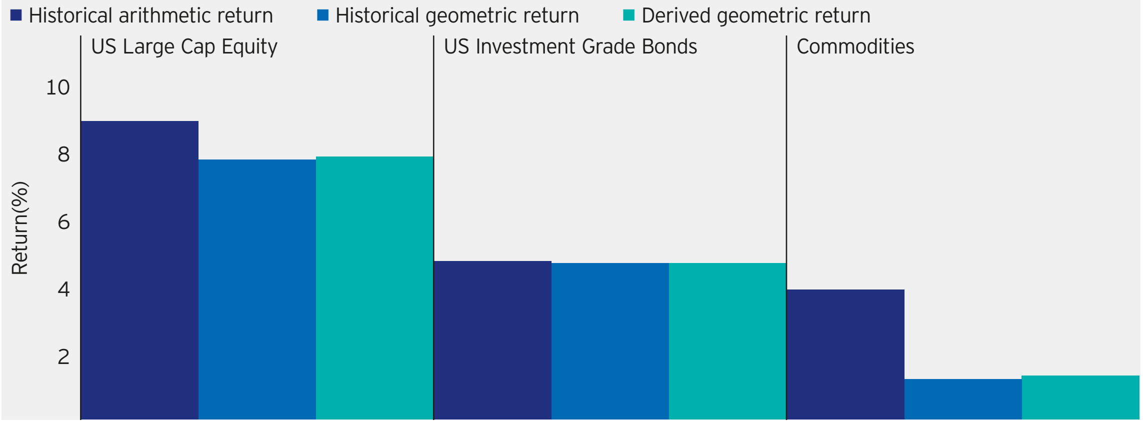
This is particularly important for portfolio construction as it describes the probability-weighted return outcome (central tendency) of a return distribution, or rather, its expected return. If the returns provided by a particular return distribution were all equally likely, then the geometric mean could serve as our expectation. However, returns for most risky financial assets are not equally likely as they exhibit some degree of variability. This variability is most commonly expressed as a function of standard deviation. It can be shown that $\mu_a > \mu_g$ when the standard deviation of a return series is greater than zero. This highlights the fact that the volatility of a return series provides a link between the arithmetic return and the geometric return. Markowitz and Blay (2013) explore various mean-variance approximations to the geometric mean and find that the following approximation provides a reasonable generalization of this relationship:

$$\mu_g = e^{\ln(1+\mu_a) - \frac{\frac{1}{2}\sigma^2}{(1+\mu_a)^2}} - 1 \approx \mu_a - \frac{1}{2}\sigma^2$$

This approximation allows investors to go back and forth between arithmetic and geometric returns as long as they know an asset's or portfolio's arithmetic mean μ_a and volatility σ. It should be noted that using the historical information (e.g., arithmetic means, standard deviations, and correlations) in a portfolio analysis will produce portfolios that will have likely performed well in the past. Expected returns should represent expectations for returns that are likely to be achieved in the future expressed in arithmetic terms. The approximation above can also be helpful in producing expected return estimates that are appropriate for use in a portfolio analysis as well as being aligned with intuition in geometric terms.

As an example of how well such a simple approximation can work, in Figure 26 we consider the historical arithmetic and geometric returns for three standard asset classes: 1) US Large Cap Equity, 2) US Investment Grade Bonds, and 3) Commodities and compare the historical geometric return with one derived from the approximation above. The two geometric returns are very close and differ by no more than 10.5 basis points in this example.

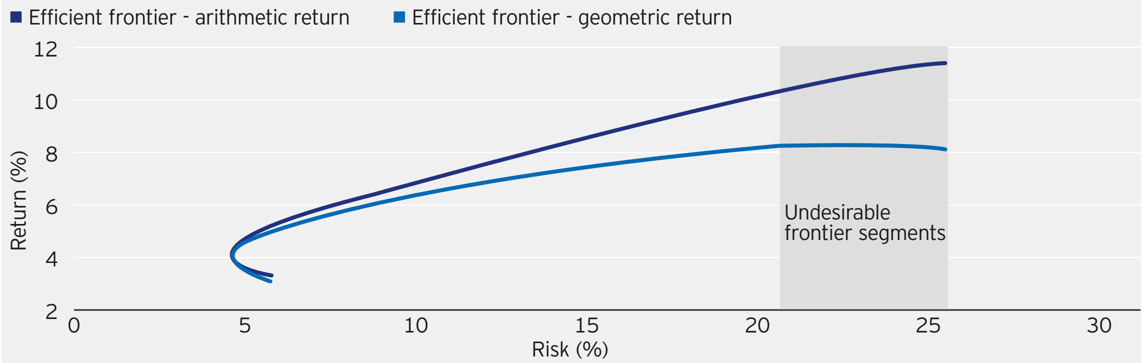
Figure 26: Historical arithmetic, geometric and derived geometric returns for select asset classes



Monthly return data period from Sept. 1, 1998 to Aug. 31, 2018.
Note: The historical volatilities of the asset classes over the period are as follows: US Large Cap Equity 14.5%, US Investment Grade Bonds 3.5% and Commodities 22.5%. Past performance cannot guarantee comparable future results.
Source: Invesco, Bloomberg.

The ability to effectively translate arithmetic returns to geometric returns (and vice versa) is of consequence to investors as the return inputs, or expected returns, used in a mean-variance portfolio optimization must necessarily be expressed in arithmetic terms. The reason for this is that the arithmetic mean of a weighted sum (e.g., a portfolio) is the weighted sum of the arithmetic means (of the portfolio constituents). This does not hold for geometric returns. In other words, the weighted average of the arithmetic means of the assets included in a portfolio is equal to the arithmetic mean of the portfolio as a whole. This is not the case when geometric means are used. Since the expected return inputs of a portfolio analysis are required to be in arithmetic terms, the outputs of such an analysis are also in arithmetic terms and must be translated, through the use of the portfolio mean and standard deviation, into the more intuitive geometric terms that describe the expected growth rates provided by the efficient set of portfolios for portfolio selection. Invesco Vision allows for more intuitive portfolio selection by presenting efficient frontiers in geometric terms. Figure 27 presents an example of an efficient frontier presented in both arithmetic and geometric terms.

Figure 27: Efficient frontier presented in arithmetic and geometric terms



Source: Invesco.

Note that the efficient frontier expressed in terms of arithmetic returns sits well above the efficient frontier expressed in terms of geometric returns. This is so because the geometric returns are downward adjustments of the arithmetic returns. It is only when we view the efficient frontier expressed in this fashion that we can see how, at segments of the frontier where portfolio volatility is sufficiently large, pursuing portfolios with higher arithmetic returns can result in the likelihood of achieving lower long-term (geometric) returns than portfolios with lower risk. Investors should avoid these segments of the frontier.

Currency adjusted expected returns

Portfolios of an international or global nature will likely invest in financial instruments that are based in foreign currencies. For instance, a UK-based multi-asset portfolio manager will likely have an appreciable allocation to US large-cap equities based in USD. Since the UK-based manager wishes to consider their portfolio returns in terms of the local GBP currency there is need to convert the forecasted returns for the US large-cap equity asset class from a USD-based perspective to a GBP-based perspective, especially for the purposes of optimal portfolio construction via mean-variance optimization or its robust counterpart.

For the example UK-based portfolio manager, given an annualized expected return of μ_{USD} for the USD-based large cap equities, and an annualized US government bond yield of i_{USD} and a similar annualized UK government bond yield of i_{GBP}, our formulation for the annualized expected return in GBP is:

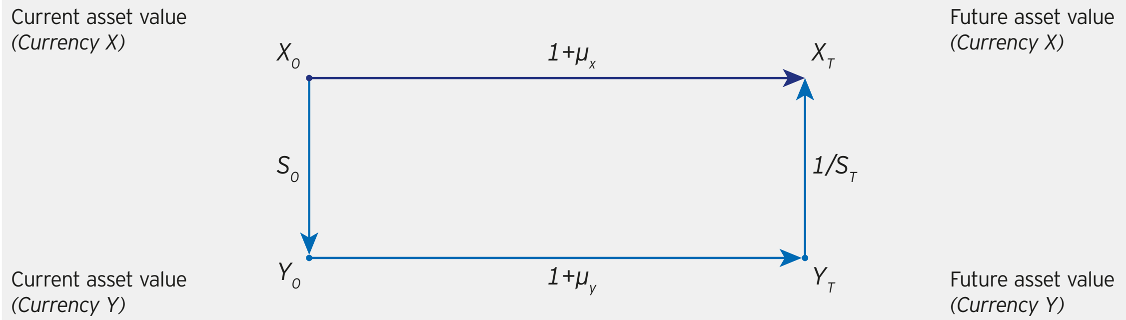
$$\mu_{GBP} = \mu_{USD} - i_{USD} + i_{GBP}$$

In what follows below, we provide the rationale for this return conversion.

At the core of our currency-based expected return conversion process is the concept of Interest Rate Parity. We utilize the basic concept that the future value of an asset denominated in currency X is equivalent to the foreign exchange rate-converted future value of the asset denominated in currency Y. Figure 28 below graphically depicts such an equivalence.

Specifically, let X_0 denote the current value of an asset denominated in currency X and let X_T denote its future value. Then, assuming a single period return of μ_x the future value is simply $x_T = (1+\mu_x)x_0$. (This is the top dark blue segment in Figure 28.)

Figure 28: Interest rate parity commutation diagram



Source: Invesco, BarraOne.

An alternative to going directly from the current value X_0 to the future value X_T (in terms its return μ_x in currency X) is to first convert the value of X_0 in currency X to the value Y_0 in currency Y. Such a conversion may be simply expressed as $Y_0 = S_0 X_0$, where S_0 is the current foreign exchange rate in going from currency X to currency Y. (This is the left-most segment of Figure 28.) Next, assuming a single period return of μ_y, the future value in currency Y is simply $y_T = (1+\mu_y)y_0$. (This is the bottom segment of Figure 28.) Finally, the future value Y_T may be converted to the future value X_T through a similar foreign exchange rate conversion. Namely, $x_T = y_T/S_T$ where $1/S_T$ is the future foreign exchange rate going from currency Y to currency X. (This is the right-most segment of Figure 28.)

Since the future value of the asset denominated in currency X should be the same as the foreign exchange rate-converted future value of the asset denominated in currency Y, so as to not violate arbitrage conditions, this means:

$$x_T = x_0(1+\mu_x) = S_0 x_0 (1+\mu_y)(1/S_T)$$

If we perform the same analysis along the same paths, now in terms of two government bonds (whose returns we treat as certain), one denominated in currency X with yield i_x and the other in currency Y with yield i_y, then we will have:

$$\frac{1+\mu_x}{1+\mu_y} = S_0/S_T = \frac{1+i_x}{1+i_y}$$

Noting that $(1+\mu_x)(1+\mu_y)^{-1} \approx 1+\mu_x - \mu_y$, and similarly that $(1+i_x)(1+i_y)^{-1} \approx 1+i_x - i_y$, means

$$\mu_y = \mu_x - i_x + i_y$$

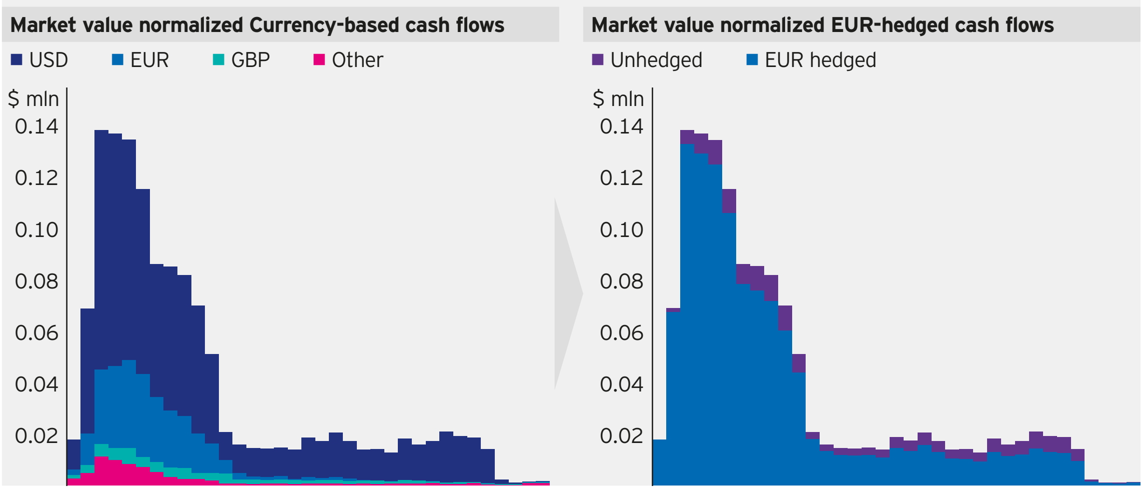
Since our portfolio construction perspective is a strategic, long-horizon one, we use the annualized yields of the 10-year government bonds in currencies X and Y in the above return conversion formula and combine them with the annualized forecasted return in currency X. This is our estimate of the forecasted annualized return in currency Y. This modeling assumption leads to similar return estimates whether we choose to hedge or not. Of course, from a risk perspective currency hedging will have a meaningful impact.

Cash flow currency translation

International, or global portfolios, will likely hold investment positions in assets that are denominated in various foreign currencies. For the case of fixed income assets, whose role may be to provide predictable cash flows, this may introduce unwanted risks due to currency and relative interest rate volatility. To address these risks, investors may choose to hedge out the currency exposure using derivatives.

While there are many ways this hedging exercise can be pursued, we assume that it is done using cross currency swaps. Figure 29 provides an example of the result of converting the cash flows provided by the Bloomberg Barclays Global Corporate Index into EUR hedged cash flows.

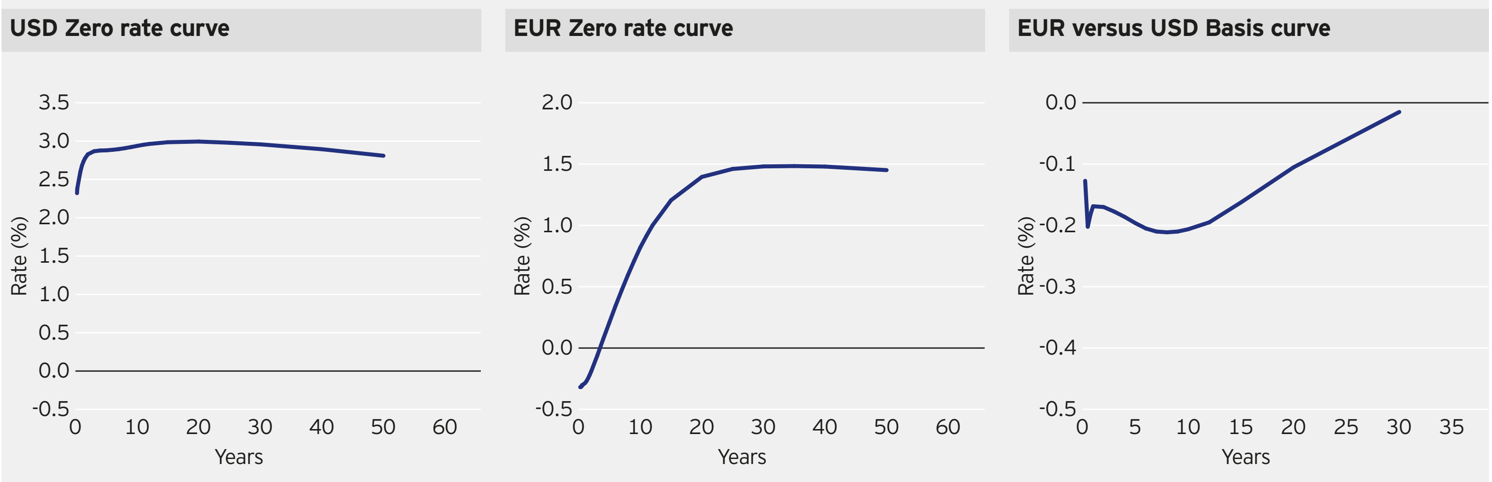
Figure 29: EUR-hedged equivalent cash flows



Source: Invesco, Bloomberg.

Conversion between any two currencies requires the use of three distinct curves. Figure 30 shows the three rate curves used for the case of converting a USD-based cash flow into EUR-based cash flows. As a first step we use the USD Zero rate curve to convert fixed USD into floating USD cash flows. We then convert those floating cash flows to fixed EUR cash flows employing the EUR Zero rate curve combined with the basis curve.

Figure 30: Discount curves used for cash flow conversion between USD and EUR



Source: Invesco, Bloomberg.

Investment risks

The value of investments and any income will fluctuate (this may partly be the result of exchange rate fluctuations) and investors may not get back the full amount invested.

Diversification and asset allocation do not guarantee a profit or eliminate the risk of loss.

Invesco Investment Solutions (IIS) develops Capital Market Assumptions (CMAs) that provide long-term estimates for the behavior of major asset classes globally. The team is dedicated to designing outcome-oriented, multi-asset portfolios that meet the specific goals of investors. The assumptions, which are based on 5- and 10-year investment time horizons, are intended to guide these strategic asset class allocations. For each selected asset class, IIS develop assumptions for estimated return, estimated standard deviation of return (volatility), and estimated correlation with other asset classes. Estimated returns are subject to uncertainty and error, and can be conditional on economic scenarios. In the event a particular scenario comes to pass, actual returns could be significantly higher or lower than these estimates.

This information is not intended as a recommendation to invest in a specific asset class or strategy, or as a promise of future performance. Refer to the IIS CMA methodology paper for more details.

Important information

All information is sourced from Invesco, unless otherwise stated. All data as of April 15, 2019 and is USD and hedged unless otherwise stated.

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