Portfolio analytics

Evaluating factor exposures
We can decompose the risks within a selected portfolio into various underlying factors. Invesco Vision includes multiple factor groups that allow investors to identify and evaluate the various risks with which they are faced. Figure 36 presents the economic factor groups considered and Figures 37 and 38 present the Solvency II and NAIC factor groups. The following sections provide detail on how to aggregate or decompose factor risks to various levels of granularity or relevant groupings.

Isolated risk and contribution to risk
Risk can be decomposed and viewed either in isolation or in terms of contribution to total risk. In both cases, we need to first define the binary group inclusion matrix M_G. It is a bit-mask-like diagonal matrix whose entries are either a 1 (meaning inclusion in the group of interest) or 0 (meaning the factor is not included in the group of interest). As an example, consider the "Rates" risk factor group matrix M_G as detailed in the group factor map shown in Figure 35.

Figure 35: The "Rates" group matrix M_G

	Factor covariance matrix								
	$Factor_1$	**$Factor_2$**	**...**	**Shift**	**Twist**	**Butterfly**	**...**	**$Factor_{k-1}$**	**$Factor_k$**
$Factor_1$	0								
$Factor_2$		0							
⋮			⋱						
Shift				1					
Twist					1				
Butterfly						1			
⋮							⋱		
$Factor_{k-1}$								0	
$Factor_k$									0

Source: Invesco, BarraOne.

With M_G defined and Σ_f, the factor covariance matrix, we can compute the isolated risk for a portfolio with B factor exposures and w weights as follows:

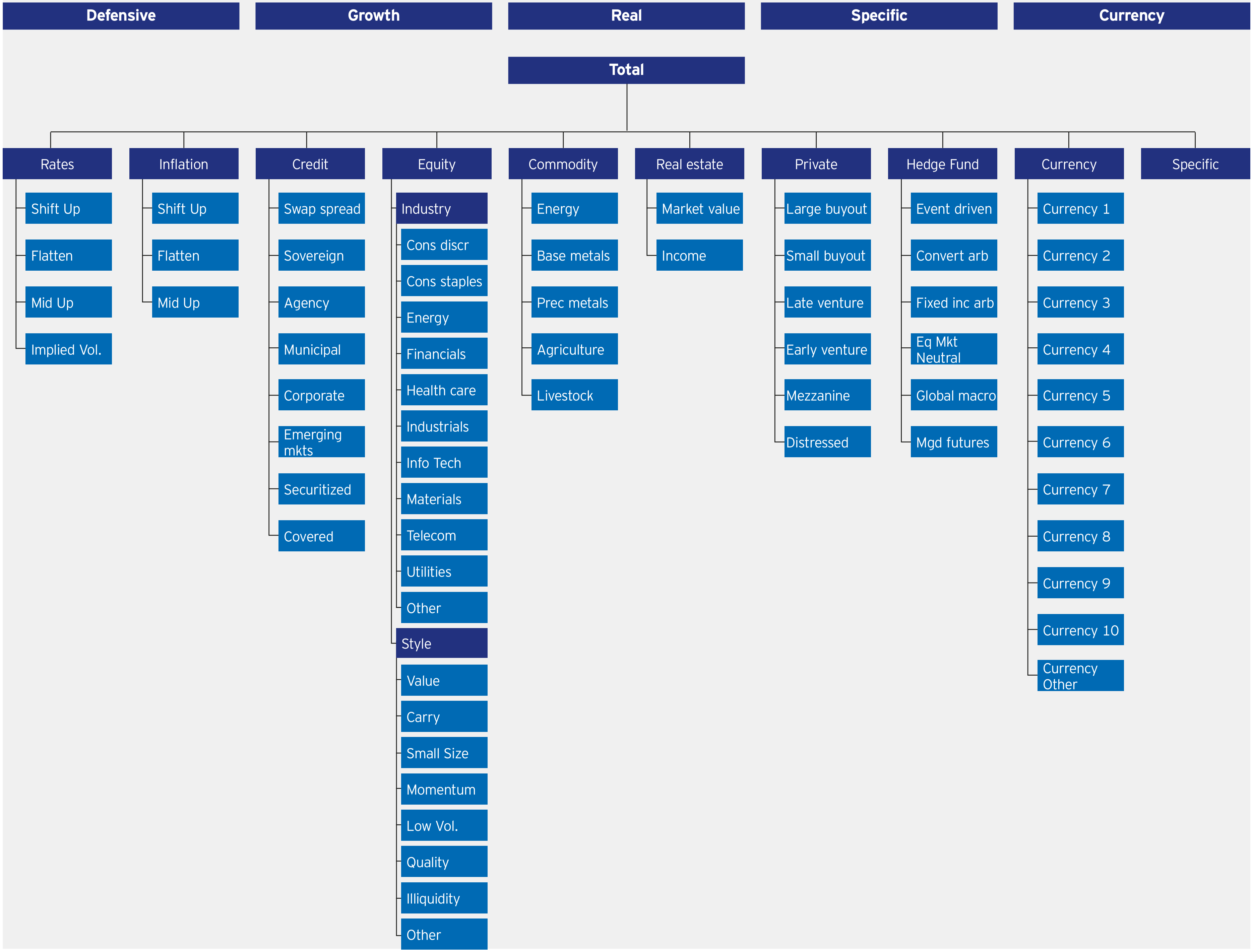
$$\sigma_{Group\ Isolated\ Risk} = \sqrt{w^T B^T M_G^T \Sigma_f M_G B w}$$

Similarly, we can calculate the contribution to risk as follows:

$$\sigma_{Group\ Conrtibution\ to\ Risk} = \frac{w^T B^T M_G^T \Sigma_f Bw}{w^T B^T \Sigma_f Bw + \sigma_{spec}^2}$$

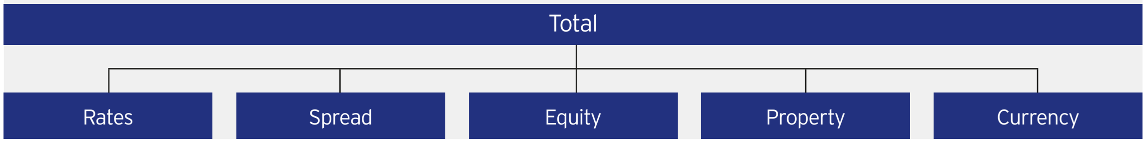
Here, the group contribution to risk is a percentage of the normalized total portfolio risk, so that all of the group contributions to risk along with the specific risk add up to 100 percent.

Figure 36: Economic factor risk groups



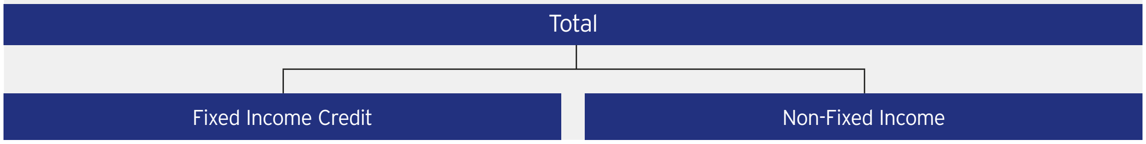
Source: Invesco, BarraOne.

Figure 37: Solvency II factor group



Source: Invesco, Solvency II directive.

Figure 38: RBC factor groups



Source: Invesco, NAIC.

Example: Group factor analysis

We consider a simple example of our factor exposure analysis for a single fixed income instrument, namely an investment grade bond issued by a telecommunications firm. In this case, there are only eight factor exposures as shown below, and the specific risk of the bond is 7.97%.

$$B = \begin{pmatrix} 15.09\ (GOV_{SH}) \\ 13.49\ (GOV_{TW}) \\ 8.53\ (GOV_{BU}) \\ 15.09\ (SWP_{SH}) \\ 11.87\ (SWP_{TW}) \\ 8.80\ (SWP_{BU}) \\ 15.10\ (TEL_{BBB}) \\ 1\ (CUR_{USD}) \end{pmatrix}$$

Assuming that we wish to focus on the interest rate risks (indicated as GOV above), we first proceed to create the bit mask as shown below. Factor entries corresponding to interest rate exposures are indicated with a 1 while the corresponding group factor covariance matrix is shown in Figure 39.

$$M_G = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

Figure 39: Factor covariance matrix Σ_f (values X 10^{-6})

	GOV_{SH}	GOV_{TW}	GOV_{BU}	SWP_{SH}	SWP_{TW}	SWP_{BU}	TEL_{BBB}	CUR_{USD}
GOV_{SH}	50.63	6.82	-6.61	-2.00	1.45	0.06	-24.18	0.00
GOV_{TW}	6.82	9.22	1.67	-1.16	-1.65	-0.75	1.27	0.00
GOV_{BU}	-6.61	1.67	3.35	-0.27	-0.56	-0.76	3.82	0.00
SWP_{SH}	-2.00	-1.16	-0.27	5.57	0.79	0.23	-2.55	0.00
SWP_{TW}	1.45	-1.65	-0.56	0.79	2.91	0.42	-7.78	0.00
SWP_{BU}	0.06	-0.75	-0.76	0.23	0.42	1.25	-1.66	0.00
TEL_{BBB}	-24.18	1.27	3.82	-2.55	-7.78	-1.66	122.94	0.00
CUR_{USD}	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00

Source: Invesco.

Putting the basic ingredients together, we calculate both isolated interest rate risk and interest rate contribution to risk as follows:

$$\sigma_{Group\ Isolated\ Risk} = \sqrt{w^T B^T M_G^T \Sigma_f M_G B w} = 12.21\%.$$

$$\sigma_{Group\ Conrtibution\ to\ Risk} = \frac{w^T B^T M_G^T \Sigma_f B w}{w^T B^T \Sigma_f B w + \sigma_{spec}^2} = 25.80\%.$$

Historical and hypothetical scenario analysis

An understanding of how a portfolio might have performed historically during various geopolitical and economic environments, as well as how it might perform in certain hypothetical scenarios that could occur in the future, can be used to inform decisions regarding the navigation of potential future market dynamics. Invesco Vision allows for these types of analyses providing detailed decompositions that help to identify key drivers of risk within a portfolio.

- **Historical scenario analysis**
  Modeling a portfolio's performance during a historical period of interest is a straightforward exercise. In general, if one considers a historical period of interest, during which the model risk factors have returns of say r_{f1}, r_{f2}, through r_{fk}, then using the known current exposures each asset has to each of the factors $\beta_{j,k}$, and the weights of each of the assets within the portfolio W_j, the portfolio's return for the period can be computed as follows:

$$r_P = \sum_{j=1}^{N} \sum_{k=1}^{K} w_j \beta_{j,k} r_{f_k}$$

It is important to note that this calculation relies on current factor exposures and that these values could have been different during any actual historical period. Historical scenario analysis provides valuable insight and yields the magnitude and direction of the portfolio's return during periods of interest. Invesco Vision covers various pre-determined historical scenarios of interest during which a user can analyze their portfolio and observe its performance, including the 1970s oil crisis, the 1987 market crash, the Global Financial Crisis, and more recently Brexit.

Example: historical scenario

We consider a simple historical example for a portfolio holding a single fixed income instrument as we did in the previous section. Here, we examine the 2010 European Bond Crisis spanning March 14 - May 26. During this time, we record the relevant factor returns as well as the corresponding telecommunications firm bond factor exposures as shown in Figure 40.

Figure 40: The factor exposures, factor shocks and factor returns
2010 European Bond Crisis using telecommunications firm bond 4.682 '46

Factors	Exposures	Shocks	Returns (exposure x shock)
GOV_{SH}	15.09	0.35%	5.27%
GOV_{TW}	13.49	0.19%	2.62%
GOV_{BU}	8.53	-0.02%	-0.20%
SWP_{SH}	15.09	0.03%	0.51%
SWP_{TW}	11.87	0.00%	0.00%
SWP_{BU}	8.80	0.00%	0.00%
TEL_{BBB}	15.10	-0.23%	-3.42%
CUR_{USD}	1.00	0.00%	0.00%
Total	-	-	**4.79%**

Source: Invesco, Barra One.

Based on the equations shown previously, the portfolio return can be computed as the sum-product of the factor exposures and the factor shocks, which is 4.79%. It is impor tant to note that this calculation is linear in nature and does not include any second order pricing effects such as convexity. When we account for such effects, the return is estimated to be 4.48%.

- **Hypothetical scenario analysis**
  Hypothetical portfolio analysis models various market movements that potentially could happen in the future. There are two types of scenarios: uncorrelated, where changes are isolated to a some specific factor, and correlated, where changes in factors are propagated across all other factors.
  - **Uncorrelated scenario analysis**
    In an uncorrelated scenario analysis with β factor exposures, w weights, and where we assume M factors are shocked by *v*, the portfolio return can be computed as follows:

$$r_P = \sum_{j=1}^{N} \sum_{k=1}^{M} w_j \beta_{j,k} r_{f_k}$$

  - **Correlated scenario analysis**
    At the heart of correlated scenario analysis is the concept of conditional expectation. We assume that our risk factor returns are distributed as a multivariate normal distribution and that the factor returns are correlated as indicated by the factor covariance matrix. In this case the basic recipe is to 1) prescribe a list of factor shocks, 2) propagate the factor shocks across the remaining factors, and 3) compute the portfolio returns as the sum-product of the factor exposures and the factor shocks.

To propagate the shock, we employ the following calculation:

$$r_f = \Sigma_{2,1} \, \Sigma_{1,1}^{-1} \, r_{f_s}$$

where r_{fs} are the originating factor shocks that are propagated to all factors r_f. $\Sigma_{2,1}$ is the covariance between the shocked factors and all other factors and $\Sigma_{1,1}$ is the covariance of the shocked factors as shown in Figure 41.

Figure 41: The factor covariance matrix block form

Factor covariance matrix blocks

	F_1	F_2	F_3	F_4	…	F_{K-1}	F_K
F_1	$\Sigma_{1,1}$		$\Sigma_{1,2}$				
F_2							
F_3	$\Sigma_{2,1}$		$\Sigma_{2,2}$				
F_4							
⋮							
F_{K-1}							
F_K							
	Shocked factors		⟵ All other factors ⟶				

Source: Invesco, BarraOne.

With all factor shocks having been propagated we can proceed to compute the portfolio returns just as we did in the historical case.

Invesco Vision allows users to consider a collection of hypothetical shocks. The shocks include movements to global equities, US equities, and EAFE equities, US Treasuries, currency exchange rates, oil, and gold. These can be viewed in correlated and uncorrelated terms.

Example: Hypothetical scenario (uncorrelated and correlated)
Using the same single asset portfolio in the above examples, we consider the hypothetical scenario in which we shock the US Treasury curve by a parallel 100-basis-point upward movement. As before, there are eight non-zero factor exposures in this example.

In the uncorrelated case, the only factor exposure movements are the ones we have shocked, GOV_{SH}, GOV_{TW}, and GOV_{BU}, and all of the remaining factor shocks are identically zero. In the correlated case, the three explicit interest rate shocks propagate and give rise to non-zero factor shocks across the remaining five risk factors. All of these shocks, along with the factor exposures, are summarized in Figure 42.

Figure 42: The factor exposures and the uncorrelated and correlated (propagated) factor shocks

			Uncorrelated		Correlated
Factors	**Exposures**	**Shocks**	**Returns**	**Shocks**	**Returns**
GOV_{SH}	15.09	-1.00%	-15.09%	-1.00%	-15.09%
GOV_{TW}	13.49	0.00%	0.00%	0.00%	0.00%
GOV_{BU}	8.53	0.00%	0.00%	0.00%	0.00%
SWP_{SH}	15.09	-	-	0.05%	0.77%
SWP_{TW}	11.87	-	-	-0.08%	-0.93%
SWP_{BU}	8.80	-	-	0.04%	0.34%
TEL_{BBB}	15.1	-	-	0.63%	9.44%
CUR_{USD}	1.00	-	-	0.00%	0.00%
Total	**-**	**-**	**-15.09%**	**-**	**-5.46%**

Source: Invesco, BarraOne.

Similar to the previous result, the portfolio level return is just the sum-product of the factor exposures and factor shocks. The portfolio return for the uncorrelated and correlated shocks are -15.09% and -5.46%, respectively. When we include full re-pricing that includes the impact of convexity, the returns are -13.58% and -5.26%, respectively.

Investment risks

The value of investments and any income will fluctuate (this may partly be the result of exchange rate fluctuations) and investors may not get back the full amount invested.

Diversification and asset allocation do not guarantee a profit or eliminate the risk of loss.

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This information is not intended as a recommendation to invest in a specific asset class or strategy, or as a promise of future performance. Refer to the IIS CMA methodology paper for more details.

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