Invesco

Risk & Reward

Research and investment strategies

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Andrew Schlossberg
President and CEO
of Invesco Ltd.

Finance and investment are constantly evolving – and the latest advancements draw on the results achieved in the past. The new edition of Risk & Reward reflects this emergent continuum by reporting on our most recent progress in responsible investing while sharing memories about Nobel Laureate and father of modern finance, Harry M. Markowitz, who passed away on June 22nd at the age of 95.

We start with an ESG perspective on buy-and-maintain strategies in fixed income. As our experts demonstrate, even with 'net zero committed' issuers, the sustainability profile of a fixed income portfolio can be improved substantially without materially harming return expectations. This is certainly good news for everyone who, like us at Invesco, cares about the impact of the companies they invest in – without forgetting that healthy returns are what drive positive change.

Taking time to look back is equally important. Our Head of Research, Kenneth Blay, has contributed a special article, paying tribute to his friend, the legendary Harry M. Markowitz – with whom he had the privilege to work and even publish a book. The 'Father of Modern Portfolio Theory' gave us the insights needed for such tools as risk-return optimization, the Capital Asset Pricing Model, factor investing, fair value analysis, and many other central elements of modern finance.

As asset managers and investors move into new frontiers – including novel ESG approaches seeking to merge the desire for sustainability with the need for sufficient risk-adjusted returns – Markowitz and his work remain as relevant as ever.

Join us in celebrating his legacy in this edition of Risk & Reward!

Best regards,

Andrew Schlossberg
President and CEO of Invesco Ltd.

Solving for sustainability

By Matthew Chaldecott, CFA®

ESG continues to increase in importance for many investors. But what about the potential impacts of ESG approaches on return and risk? For buy-and-maintain strategies in fixed income, we find that the sustainability profile of a portfolio can be improved substantially without materially impacting return expectations. Furthermore, as many investors aim for net zero, we explore frameworks for monitoring and measuring decarbonization progress.

Sustainability considerations are indeed becoming more and more important: At the end of 2021, 3,800 investors accounting for USD 121 trillion in assets had signed the UN Principles for Responsible investment – up from fewer than 1,000 investors (USD 24 trillion) a decade earlier.[1] The Net Zero Asset Owners Alliance has also grown to 86 entities representing a total of USD 11 trillion,[2] and the Net Zero Asset Managers initiative has grown to over 300 signatories with USD 59 trillion in assets under management.[3]

Despite this trend, there is a stubborn preconception among some market participants that ESG-oriented investing is synonymous with lower returns. Their conviction was only strengthened with the start of Russia's war in Ukraine, when the spike in oil & gas prices led to soaring profits for many energy companies. Indeed, if we compare the MSCI World Energy Index (+48% in 2022) with the overall MSCI World Index (-18% in 2022), we see a performance gap of 65 percentage points.[4]

SRI and ESG indices trade in line with the main universe, and have done so historically.

And what about credit? In 2022, the Bloomberg Global Aggregate Corporate Energy Index actually slightly underperformed the main index (-14.73% vs. -14.11%, figure 1). Admittedly, this has a lot to do with the longer duration of the energy index in a year when Treasury yields rose by around 2 percentage points.[5] In the same year, the SRI index, which excludes issuers involved in controversial sectors and activities, fell by 13.85%; the ESG index, which weights issuers according to their ESG scores, fell by 13.60%. This aligns with research showing that, once factor exposures are matched, return patterns are very similar – and there is even potential for a positive ESG premium.[6]

Bringing this back to the equities example, we can also think about the factor exposure of the two indices. The energy sector has historically offered a higher dividend yield compared to the main universe – at the end of 2021 the rates were 4.37% vs. 1.67% respectively. If we think of the two as perpetual bonds and use the formula (1+y)/y to estimate the duration, the energy index comes out at 24y while the main index has a rough duration of 61y. The performance difference for 2022 is therefore more than explained [2% x (61-24)] by mathematical duration effects!

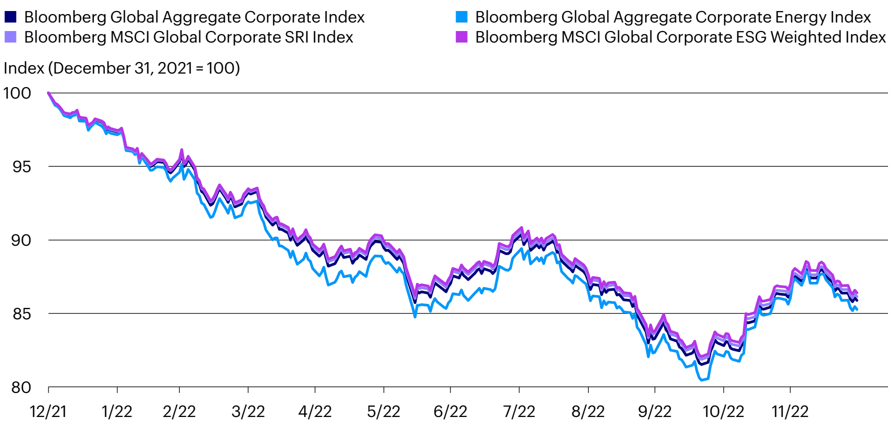
The yields-to-worst confirm this (figure 2). Again, the SRI and ESG indices trade in line with the main universe, and have done so historically. The maximum divergence over the past five years has been 13 basis points.

But, as with financial fundamentals, the momentum and direction of travel can matter as much as the current yield. Indeed, studies have found evidence of a higher information ratio for companies with positive ESG trajectories.[7]

Net zero: The new frontier

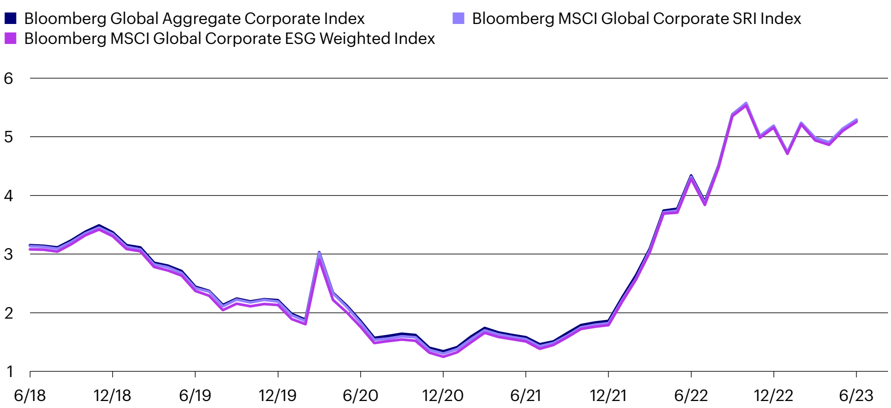
As many companies and countries seek to achieve net zero emissions by 2050, decarbonization has emerged as one of the most important sustainability topics. According to our research, around 2/3 of the global corporate universe (by market value) are "net zero committed". This is still a huge investible universe of nearly 10,000 securities with an aggregate market value of over USD 10 trillion.[8] The yields and spreads of this net zero universe are very close to the full universe – the average

Figure 1
No major performance differences in 2022



Source: Bloomberg. Based on total returns in USD from December 31, 2021 to December 31, 2022. All indices are Total Return, hedged in USD. **Past performance does not predict future results.** An investment cannot be made in an index.

Figure 2
Yields-to-worst have been similar

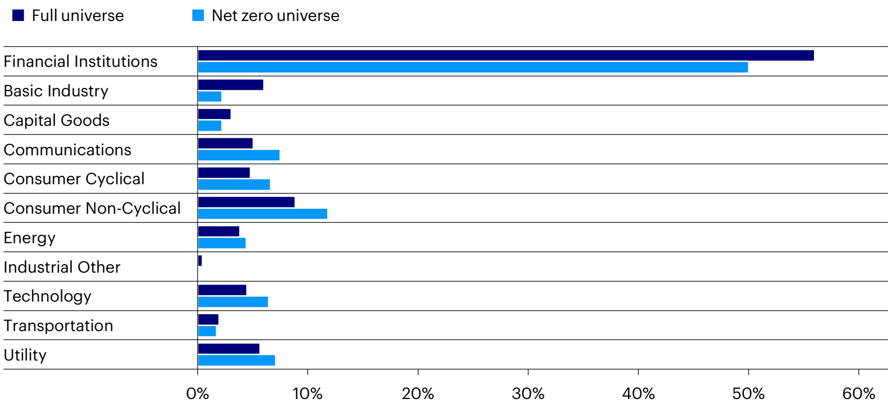


Source: Bloomberg. As of June 30, 2023. An investment cannot be made in an index.

The yields and spreads of this net zero universe are very close to the full universe

Figure 3
Analyzing the global corporate bond landscape

Characteristic	Full universe	Net zero universe
Average credit rating	BBB-	BBB-
Market value (USD)	17,270 bn	10,198 bn
Number of issuers	2,149	820
Number of securities	15,658	9,973
Duration (option adjusted)	6.5 years	5.8 years
Spread (OAS vs. gov, USD term hedged)	158 bps	138 bps
Yield-to-worst (USD 3m hedged)	5.78%	5.47%
Yield-to-worst (term hedged)	5.63%	5.49%



Source: Invesco. As of June 30, 2023. Full universe = Bloomberg Global Aggregate Corporate Index. An investment cannot be made in an index.

credit rating is the same and the industry breakdown is very similar (figure 3).

Solving for multiple dimensions

In the past, most investors had two dimensions to think about – risk and return. Today, the landscape is becoming ever more complex, with often conflicting considerations, such as sustainability, regulatory, liquidity and liability-matching, gaining prominence. In this section, we use a four-step process to optimize portfolios for yield while satisfying constraints relating to net zero and other sustainability parameters.

Step 1 – Defining the universe

To name just a few possible parameters, investors may want to invest solely in their domestic currency, in corporate bonds or in investment grade-rated securities. Further screens are often required – in Invesco's European SFDR Article 8 portfolios, for example, issuers with material involvement in controversial activities, including tobacco, weapons or human rights violations, are excluded.[9] Investors may have additional criteria they want to impose, be it a minimum overall ESG rating or minimum individual pillar scores.

The impacts of such exclusions will vary depending on the starting universe. In general, they are modest for developed market and investment grade issuers, while for high yield and/or emerging market universes the impacts become more meaningful (figure 4). Interestingly, even though the drop off in market value or issuer numbers can be significant, the yield differentials are small.

Figure 4
Indices and universes in comparison

	Bloomberg Global Aggregate Corporate Index		ICE BofA ML US High Yield Index		JP Morgan CEMBI Broad Diversified Index	
	Full universe	Article 8 screened	Full universe	Article 8 screened	Full universe	Article 8 screened
Number of issuers	2,149	1,599	892	592	763	566
Number of securities	15,658	13,300	1,869	1,398	1,873	1,556
Market value (USD)	17,270 bn	15,004 bn	1,223 bn	935 bn	1,809 bn	932 bn
Yield-to-worst (USD hedged)	5.78%	5.67%	8.55%	8.21%	7.52%	7.32%

Source: Invesco. As of June 30, 2023. An investment cannot be made in an index. "Article 8 screened" refers to the screening process applied at Invesco. An Article 8 Fund under SFDR is defined as "a Fund which promotes, among other characteristics, environmental or social characteristics, or a combination of those characteristics, provided that the companies in which the investments are made follow good governance practices".

Once the ex-ante universe has been narrowed down, we need to define limits and targets.

Step 2 – Set constraints and optimize

Once the ex-ante universe has been narrowed down, we need to define limits and targets. We set thresholds for rating, sector weights or any combination of variables based on a custom coding function. These thresholds may be at the overall portfolio level (such as total carbon emissions) or the micro level (e.g., BBB-rated securities capped at 0.5% weight per issuer). We may also want to program in a cashflow obligation schedule for a defined benefit pension fund or an insurance company's claim expectations.

We then seek to construct the portfolio satisfying all these conditions. We may seek to optimize the output in local yield (for domestic portfolios), 3-month hedged terms (for international ones taking out FX risk) and fully hedged (where long-term cross currency swaps can be used). We may also optimize purely on credit spread (government or otherwise) in cases where duration is managed separately.

As an example, we show the potential impact of tightening or loosening a constraint – in this case carbon intensity (figure 5). The optimized model has an intensity of 147t CO2 equivalents per million USD revenue, for a yield of 6.71%. However, we can see that it is possible to reduce the carbon intensity to less than 100t for a yield sacrifice of just 2 basis points. Thereafter, the trade-off gets steeper: From 100t to under 50t costs approximately 12 additional basis points of yield.

Step 3 – Qualitative review

Our approach is not a purely quantitative "black box" model. Portfolio managers assess the output qualitatively to ensure that it makes sense both in the letter and the spirit of the investment constraints and objectives. Furthermore, the initial guideline specifications may not be perfect, producing some unintended portfolio concentrations. In such cases, the model portfolio can be manually adjusted. Another possibility is to adjust the coding of the constraints before re-running the optimization process.

Step 4 – Ongoing management

Any portfolio needs to respond to changes in market conditions and issuer fundamentals. Faced with substantial in/outflows, many managers tend to pro-rate existing positions. We, however, prefer to re-scan the entire investable universe and propose a basket of trades to complement existing holdings and optimize the end result.

Decarbonization: What does success look like?

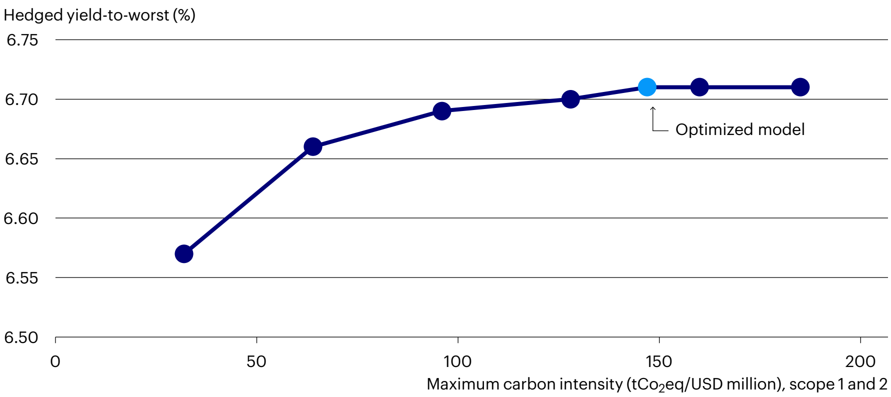
We now turn to our second topic – regular decarbonization monitoring. With more and more investors making net zero commitments, we need to monitor decarbonization progress at issuer and portfolio level. For simplicity, we assume a portfolio with just three positions, where each issuer emitted 100t of CO2 in 2020. Issuers A and B have decarbonization commitments (with B's more ambitious), while issuer C has none. Over the subsequent five years, we can compare the actual and forecast emissions to the commitment schedule.

In our example, the portfolio has been decarbonized by just 4% by the end of 2022 and lags the target of 6% by 2 percentage points. Furthermore, the gap is expected to worsen to 5.3 percentage points by the end of 2024.

To analyze the sources of this miss, we compare each issuer's commitment to its actual reduction (figure 7). It turns out that, despite reducing emissions by more than the other two, issuer B is the primary culprit for not meeting the target – simply because this company was over-ambitious in its goals. The analysis also indicates that 2023 is a critical year for decarbonization commitments.

Such data helps to flag issuer B as a priority candidate for engagement – the conversation would revolve around the ability and willingness of the issuer to get back on track, or that expectations will instead need to be revised. Beyond any sustainability objective for the portfolio, there may be investment performance impacts as markets notice a lack of

Figure 5
Which combination of yield-to-worst and carbon intensity is optimal?



Source: Invesco. As of October 2022. For illustrative purposes only. Yield in GBP-hedged terms.

Figure 6
Comparing commitments with actual emissions: an example

		2020	2021	2022	2023	2024
Decarbonization commitment (tCO_2)	Issuer A	100	97	94	91	88
	Issuer B	100	93	86	79	72
	Issuer C	100	101	102	103	104
	Total portfolio	**300**	**291**	**282**	**273**	**264**
	Cumulative commitments		**-3.0%**	**-6.0%**	**-9.0%**	**-12.0%**
Emissions (tCO_2)	Issuer A	100	98	96	94 (f)	92 (f)
	Issuer B	100	95	90	90 (f)	88 (f)
	Issuer C	100	101	102	103 (f)	100 (f)
	Total portfolio	**300**	**294**	**288**	**287 (f)**	**280 (f)**
	Cumulative decarbonization		**-2.0%**	**-4.0%**	**-4.3% (f)**	**-6.7% (f)**

Source: Invesco. Hypothetical example, for illustrative purposes only. (f) = forecast (shaded in blue).

Figure 7
Who is the culprit?

		2020	2021	2022	2023	2024
Commitment minus emissions (tCO_2)	Issuer A	0	-1	-2	-3	-4
	Issuer B	0	-2	-4	-11	-16
	Issuer C	0	0	0	0	4
	Total portfolio	**0**	**-3**	**-6**	**-14**	**-16**

Source: Invesco. Hypothetical example, for illustrative purposes only.

progress, and investors may demand a higher risk premium or choose to divest.

Conclusion

Contrary to lingering perceptions, adopting a more sustainable approach does not inherently mean sacrificing returns. But it is crucial to have the right tools to manage ESG considerations on top of classic risk and reward variables. New developments allow investors to screen and solve across multiple dimensions, helping build efficient and unbiased portfolios. Nevertheless, for the ongoing management of sustainable portfolios, analysis and attribution of ESG factors is needed and can help guide engagement.

Notes
1 Source: About the PRI | PRI Web Page | PRI (unpri.org), June 2023.
2 Source: Members – United Nations Environment – Finance Initiative (unepfi.org), June 2023.
3 Source: The Net Zero Asset Managers initiative, June 2023.
4 Source: Bloomberg, total returns in USD from 31 December 2021 to 31 December 2022.
5 The performance differntial between the two equity indices can also be explained in terms of duration differences.
6 E.g., Barclays Quant Equity Americas Advisory Council (June 2023): Integrating ESG as a Constraint and for Alpha Generation in Systematic Investing.
7 E.g., Zoltán Nagy, Doug Cogan, Dan Sinnreich (February 2013): Optimizing Environmental, Social, and Governance Factors in Portfolio Construction: An Analysis of Three ESG-tilted Strategies, MSCI Research Insight.
8 As of June 2023.
9 Invesco's SFDR Article 8 portfolios exclude issuers with material involvement in UN Global Compact violations, thermal coal extraction / power generation, unconventional oil & gas extraction, controversial weapons, tobacco and cannabis. Other activities may be excluded at fund level; please consult the offering documents for details.

About the author

Matthew Chaldecott, CFA®
Senior Client Portfolio Manager
Invesco Fixed Income

Matthew Chaldecott serves as a specialist on global buy & maintain strategies, providing analysis, thought leadership and product expertise to institutional and wholesale clients.

Harry M. Markowitz: Father of modern finance

By Kenneth Blay

Harry Markowitz, widely considered the 'Father of Modern Portfolio Theory', died on June 22nd, 2023, at the age of 95. Markowitz was a visionary, a philosopher, a brilliant researcher and scholar, a Nobel laureate, a mentor, and a friend. I had the privilege of working with Harry for over a decade and have the distinct honor of being the only person to co-author a book with him. Together we advanced the theory and practice of asset allocation and became close friends. In this article, I'll share what I believe he would have wanted people to know about him and his work, as well some insights I gained from our time together.

Harry Markowitz is best known for his pioneering work in portfolio theory, for which he was awarded the Nobel Prize in Economic Sciences in 1990. However, this is only part of the story. He was also deeply involved in advancing simulation methods. In fact, one year earlier, Harry was awarded the prestigious John von Neumann Theory Prize by the Institute for Operations Research and Management Sciences for his work in portfolio selection, mathematical programming and simulation.

Harry was the right person at the right time. He benefitted from studying under and working with some of the most distinguished names in mathematics, statistics, economics and finance. This included people like James Tobin, Milton Friedman, Kenneth Arrow, Paul Samuelson and Robert Merton, to name a few. What made Harry so special was his willingness and, more importantly, his ability to innovate and constructively challenge many of the ideas presented to him by his mentors and colleagues – many of whom would also be awarded Nobel Prizes for their own contributions. Harry was a smart kid who played well with smart people.

Harry with a replica of his Nobel Prize in his office.

Much of what we understand about investing today, from portfolio construction to risk management to quantitative and factor investing, can be traced back to Harry and his ideas, a fact that is largely taken for granted. To understand the extent of his influence, we should consider what investing was like before Harry.

The world before Markowitz

Most investors assume that diversification is an indisputable principle of investment management. But this wasn't always the case: Before 1952, investing was closer to speculation than it was to the systematic practice it is today. The reigning ideas of Benjamin Graham, David Le Fevre Dodd, Sidney Cottle and John Burr Williams implied that an investor should simply invest in stocks with the highest expected returns. Even John Maynard Keynes, one of the most influential economists of the 20th century and investor, was adamant that diversification was "a travesty of investment policy."[1]

Despite such views, investors still tended to diversify in practice. However, they often applied diversification crudely, relying on haphazard planning, improvised strategies, intuition or even hunches to make their investment decisions.[2] A 1945 text aptly titled Diversification of Investments provides some insight: "An examination of some fifty books and articles on investment that have appeared during the last quarter of a century shows that most of them refer to the desirability of diversification. The majority, however, discusses it in general terms and does not clearly indicate why it is desirable."[3]

It wasn't until Markowitz that investors were given the answers to *why* they should diversify – and how they could go about it most effectively.

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1952: The birth of modern portfolio theory

To my mind, the term 'Modern Portfolio Theory' is a misnomer. Nothing that might have been considered portfolio theory, pre-modern or otherwise, existed before Harry's 1952 Journal of Finance article, "Portfolio Selection". There simply was no portfolio theory before it.[4]

This 14-page article shifted the focus of investing from selecting individual stocks to selecting individual portfolios. It cemented risk as a central component of investment decision making and forever changed the practice of investing. As financial historian Peter Bernstein noted: "It was Markowitz who first made risk the centerpiece of portfolio management by focusing on what investing is all about: investing is a bet on an unknown future (...) Nothing more deeply divides [modern finance] from the world before 1952."[5]

Harry Markowitz was only 24 years old when his article was published. As if it wasn't enough, he published two other significant papers that same year. The first, titled The Utility of Wealth,[6] helped shape what we know today as 'prospect theory', for which Daniel Kahneman received the Nobel Prize in 2002. It's the reason why Harry is also understood to be the grandfather of behavioral finance. The second paper, titled Social Welfare Functions Based on Individual Rankings,[7] countered Kenneth Arrow's 'impossibility theorem' and demonstrated that the impossible was, in fact, possible. The ideas in this paper would later be crystalized into a method for optimizing the throughput of data traffic across computer networks. Harry called 1952 his *annus mirabilis*, referencing Einstein's four papers published in 1905. Indeed, 1952 was to finance what 1905 was for modern physics.

> Harry Markowitz was only 24 years old when his article was published.

The key insight of Portfolio Selection was that investors should consider not only expected returns but also the risk of the portfolio as a whole – which depends on the covariances between portfolio investments, i.e., the way portfolio investments interact with each other. This explained why diversification wasn't simply about increasing the number of investments. Investors need the right *kind* of diversification if they want to reduce risk. Harry gave them the tools to trace out what is now known as an 'efficient frontier' of portfolios that provide the least risk for each possible level of expected return or, conversely, the highest expected return for each level of risk. Investors could now choose how much risk they would bear.

While some might say this insight amounted to nothing more than applying existing statistical methods to the problem of investing, the fact is that no one had done so until Markowitz pointed the way. As Stanford University professor Sam L. Savage noted: "The mathematics of statistical dependence was around long before Harry Markowitz and was of interest to almost no one."[8] Furthermore, the portfolio selection problem, as Harry presented it, was based on far more than a simple application of statistical methods. As his three 1952 articles indicate, he was thinking deeply about much more than just statistics. Portfolio theory was influenced by deep philosophical, mathematical and statistical insights derived from game theory and utility theory, as well as statistics.

> "The mathematics of statistical dependence was around long before Harry Markowitz and was of interest to almost no one."

A key influence on his work was Bayesian statistician, Leonard J. Savage, from whom he learned about how one should act in the face of uncertainty. Savage held that in the absence of objective probabilities, rational decision makers should use their subjective beliefs about probability to arrive at their most desirable outcomes – to maximize their expected utility, in the language of economics. Based on the ideas of Savage and others, Markowitz proposed that, while the objective probability distributions of future returns for investments are unknown, investors have (or can develop) their own subjective beliefs about the distributions of those returns. Investors should then use return expectations along with their subjective probability beliefs to diversify their portfolios in such a way as to maximize expected utility.[9] In short, While he was proud of having received the Nobel Prize, it was the von Neumann Prize that meant the most to him.

portfolio selection is about making the best use of your beliefs about the future. The first three sentences of the Portfolio Selection article make this clear: "The process of selecting a portfolio may be divided into two stages. The first stage starts with observation and experience and ends with beliefs about the future performances of available securities. The second stage starts with the relevant beliefs about future performances and ends with portfolio choice. This paper is concerned with the second stage."

1959: A masterpiece is published

Later, Harry more fully detailed the ideas on which his portfolio selection method was based when, at the invitation of James Tobin, he spent the 1955-1956 academic year working with the Cowles Foundation at Yale University. During this year, he reviewed and refined his work on portfolio theory and penned most of his book Portfolio Selection: The Efficient Diversification of Investments,[10] which would eventually be published in 1959.

Along with lessons in statistics and mathematics required for understanding portfolio selection, his book details the critical line algorithm for the derivation of efficient portfolios. In Part IV, aptly titled Rational Choice Under Uncertainty, Markowitz provides the fundamental assumptions for portfolio theory. This includes the expected utility maxim, utility analysis over time and probability beliefs. The last chapter of Part IV, and indeed of the whole book, is titled Applications to Portfolio Selection. It was of particular interest to me as a practitioner, as it includes, among other things, a discussion of various risk measures that might be considered for portfolio selection. It presents the utility functions implied by each measure of risk and details of how one might consider evaluating alternative risk measures. This demonstrates the kind of thinking that many practitioners and quantitative analysts would benefit from today as they consider portfolio construction approaches.

Virtually every serious question I've had about portfolio selection is addressed in some fashion in Part IV of the book. If it doesn't provide a specific answer, it provides guidance on how to think about the problem.

Taken as a whole, Harry's 1959 book is a masterpiece. It provides as complete an argument for portfolio selection as anyone – practitioner and academic alike – can expect. It's also easy to take for granted that the work is as much philosophy as it is mathematics and statistics – and it was completed virtually without the help of computers. In fact, Harry had hoped to include an example of a 25-security portfolio analysis in the book but realized that the programming and computing time required was beyond what was available to him at the time. Today, Markowitz's portfolio theory is taught at colleges and universities around the world. However, it seems the focus is on the mechanics of the portfolio theory rather than the fundamental assumptions that underpin its use. This means that, generally, only half the story is being told. Every serious asset allocation practitioner should read Harry's 1959 book cover-to-cover at least once to understand the full story.

Beyond models: Financial market simulation

After completing his 1959 book Portfolio Selection, Harry shifted his attention to other endeavors – in particular, the design and development of the SIMSCRIPT programming language that would facilitate the practical application of simulation and which today is used worldwide by a variety of entities, including Lockheed Martin, the US military, the Federal Aviation Administration (FAA), NASA, NATO and over 20 countries. As mentioned previously, Harry would be awarded the prestigious John von Neumann Theory Prize for his work in portfolio selection, mathematical programming and simulation. While he was proud of having received the Nobel Prize, it was the von Neumann Prize that meant the most to him.

Harry would continue to innovate with simulation, as he believed solutions to many problems lay beyond simplified models. Life and the financial markets were far too complex. For example, even though the capital asset pricing model (CAPM) was based on his ideas, he understood the limits of the model, and even published several papers explaining some of the model's shortcomings and their implications for investors.[13] Beyond his work on SIMCRIPT, he advanced the practical use of simulation on several fronts:

Financial market simulation: The work presented in Kim and Markowitz (1989) developed a market simulation that demonstrated how portfolio insurance was at the center of the 1987 market crash. This work arose out of a dispute with Fisher Black, who questioned the role of portfolio insurance in the crash. Because there was no historical market data to inform the question, Harry created it through simulation. Harry would later go on to develop the Jacobs, Levy, Markowitz simulator (JLMSim; Jacobs et al. 2004) that would allow investors to conduct their own financial market simulations. This work advanced the use of agent-based modeling in financial markets for risk management.

"The Game of Life": In "Individual versus Institutional Investing" (1991), Harry first proposed a "Game-of-Life" simulator which could provide decision rules that are more credible than those produced by analytic methods.[14] He later went on to work with GuidedChoice® to advance and implement this idea as part of a computer-assisted portfolio selection service for retirement investors. Today, portfolio management through robo-advice is well-established.

Multi-period portfolio selection: Markowitz and van Dijk (2003) presented an approximation of dynamic programming solutions allowing dynamic portfolio allocation across changing market conditions. Blay and Markowitz (2015) used simulation to address the complexities of taxation's impact on portfolio selection. And Blay, et al. (2018) used simulation to provide a flexible approach to addressing the multi-period portfolio selection problem.

As Harry advanced the use of simulation in finance, mean-variance analysis had been gaining traction in practice. However, his portfolio theory work would be challenged as a result of global events – in this case, the 2008 financial crisis. Investors began to question the efficacy of mean-variance analysis as concerns of non-normal return distributions and fat tails came to the fore. The argument being advanced was that mean-variance should not be used because it assumes return distributions are normal. But Harry never assumed return distributions were normal, nor does mean-variance require normal distributions. This "Great Confusion," as Harry called it, would ultimately lead him to revisit his 1959 work.

The Age of Portfolio Relativity: How other researchers built on Harry's ideas

One of the implications of Einstein's theory of relativity is that the passage of time is relative to the speed of the observer. The implication of Harry's portfolio theory work was that portfolio selection is relative to an investor's beliefs about portfolio investments. Central to this was minimizing portfolio risk, which depends on understanding the relationships between portfolio assets – the covariances. When writing his 1959 book, Harry understood that the data required to produce useful covariance matrices for a large number of securities did not exist, and access to computing power was highly limited. What he suggested was the possibility of developing a model of covariance based on a single-index or one-factor linear model. Then, in 1960, a young man named Bill Sharpe came to visit Harry to discuss ideas for his doctoral thesis. They both agreed on the need for models of covariance that simplified the process of producing the requisite inputs for portfolio selection.

In 1963, Sharpe published a paper titled A Simplified Model for Portfolio Selection,[11] where he introduced a one-factor model of covariance. In 1964, based on idea of considering a world full of Markowitz mean-variance investors, Sharpe followed this with the publication of Capital Asset Prices: A Theory of Market Equilibrium under Conditions of Risk,[12] which would introduce the world to the Capital Asset Pricing Model (CAPM) and the capitalization-weighted market portfolio. The key insight from this paper was that, given the assumptions used, the expected returns for assets were related to the asset's market risk, which we now know as 'beta'. Taken together, portfolio theory, models of covariance and market beta ushered in the age of portfolio relativity and transformed to the world of investing.

We can draw a straight line from Harry's fundamental insights on portfolio selection, risk and asset relationships to many of the most important breakthroughs in theoretical and practical finance since 1952, including:

Risk models: The development of Sharpe's 1963 one-factor/market index model of covariance was an early predecessor to more modern risk models developed by Elton and Gruber (1973), Rosenberg (1974) and others. Risk models are now a ubiquitous part of the asset management landscape and are broadly used for portfolio construction and risk management across both active and passive strategies.

The market portfolio: The development of the CAPM by Treynor (1962), Sharpe (1964), Lintner (1965) and Mossin (1966) introduced the world to the capitalization-weighted market portfolio and relates an asset's expected returns to its market risk, or beta. Investors thus had a method for estimating expected returns. They also had a market portfolio that could be used for comparison against other portfolios/strategies. This theory also paved the way for the development of broad market index funds.

Factor investing: Reinganum (1981), Fama and French (1992), Carhart (1997) and others identified anomalies in asset prices that countered the efficacy of the CAPM and argued for the inclusion of additional factors, such as size, value and momentum, for more effective asset pricing. These factors and others would eventually become investable factor strategies and would lead to 'smart beta' and other systematic strategies.

Performance evaluation: Treynor (1965), Sharpe (1966), Jensen (1968), and Treynor and Black (1973) developed some of the most-used performance evaluation metrics in practice today – the Treynor ratio, the Sharpe ratio, Jensen's alpha and the information ratio. These measures solidified risk and the market as central aspects of performance evaluation.

In the last 30 years, the financial services industry has increasingly embraced the theoretical and practical advances made possible by Harry's work. This has resulted in a broad transformation of asset management and financial advice that includes:

The shift from selling to advice: As the concept of asset allocation gained traction with financial advisors in the early 2000's, financial advice was transformed from the pitching of individual stocks to the offering of efficiently diversified portfolios customized to address the unique risk/return objectives of individual investors.

Investment management innovations: These include quantitative approaches to active asset management (Grinold and Khan, 1994), improved risk models that allow for better risk targeting and management, and the optimization of excess returns for more efficient use of tracking error (Chow, 1990; Waring et.al., 2000).

Product and customization innovations: As asset allocation and investment management techniques advanced, so did the products and services that are now broadly available to everyday investors. From a broad array of index-based ETFs to factor strategies to the active management of the tax implications of investing, more investment products and services are available today than ever before to facilitate efficient asset allocation and to address specific investor preferences in achieving investor objectives.

While all of this constitutes an impressive impact by any measure, it is easy to forget that we are still only a few years into the broad adoption of the ideas and methods Harry initiated. As computing power and access to useful datasets continue to improve, we are seeing asset managers position themselves to provide more customized solutions to address unique investor needs and preferences at lower cost. I believe Harry's transformation of asset management has only just begun.

Portfolio selection research with Harry.

Harry announced that he wasn't going to write a paper. Instead, he would write a four-volume book!

Harry details how there is now more than fifty years of extensive research showing that certain functions of mean and variance do quite a good job of approximating various utility functions for a variety of return distributions.

The Great Confusion: Portfolio Selection revisited

After the 2008 global financial crisis, talk of non-normal market return distributions and arguments against the use of mean-variance optimization began to emerge. The arguments were based on the fallacious belief that mean-variance should not be used because it requires normal return distributions.

In 2008, I was managing discretionary multi-asset model portfolios at a regional broker dealer focused on serving Certified Public Accountants (CPAs) who were incorporating financial services into their practice. Our CPA advisors increasingly began to reach out with questions about non-normal distributions and whether we should continue our use of mean-variance in determining our asset allocation policy. As a function of these queries, I reached out to Harry to see if he would be interested in helping us address these concerns. Harry agreed, and we began a research partnership to write a paper that countered the non-normal distribution argument, review our firm's asset allocation policy and develop a portfolio optimization methodology that considered the impact of taxes on investor wealth outcomes.

A few weeks into our partnership, Harry announced that he wouldn't write a paper. Instead, he would write a four-volume book! His idea was to revisit in detail each of the four chapters that made up Part IV of his 1959 book and discuss how mean-variance had held up in practice over the roughly 50 years since the book's initial publication. Harry had a child-like excitement about the prospect of revisiting his portfolio selection ideas. At 82 years of age, it was an ambitious endeavor, to say the least – and it would be his final say on portfolio selection. It turned out to be much more than that.

In Volume I,[15] Harry set out to accomplish two things: The first was to revisit Chapter 10 of his 1959 book and, once again, present the expected utility maxim, which is a central tenet of portfolio theory. The second was to dispel 'The Great Confusion' – namely, the confusion between the necessary and sufficient conditions for the use of mean-variance analysis in practice.

Harry opens (on the second page of the preface) clearly stating that he justifies mean-variance analysis by relating it to the theory of rational decision making over time and under uncertainty, as developed by von Neumann and Morgenstern (1944), Savage (1954) and Bellman (1957). One of the implications of this is that the rational choice among different portfolios requires identifying portfolios that maximize expected utility. In Chapter 1, Harry walks through of the theory of rational behavior and details the expected utility maxim, which describes the principles, or axioms, used by a rational decision maker to act. This makes the case for expected utility maximization. In Chapter 2, he discusses mean-variance approximations of expected utility: Rather than attempting to determine every investor's unique utility preferences, he suggests maximizing a mean-variance approximation of a utility function that represents investor preferences. As an example, he uses the log utility function, given that it is aligned with utility theory as presented by Bernoulli (1738). It also implies that investors are risk averse. What Harry explained, in the book and many times in person, is the following: If a risk-averse investor carefully chooses a portfolio from a mean-variance efficient frontier, then the investor will approximately maximize expected utility for a wide variety of concave (risk-averse) utility functions, whether or not they understand or know about the theory of rational behavior, whether or not they understand what expected utility is, and whether or not portfolio distributions are normal. The calculations of means and variances are not dependent on return distributions.

Table 1
Comparison of log utility and a mean-variance approximation

Return (%) r	Log utility $\ln(1+r)$	Mean-variance approximation $r - \frac{1}{2}r^2$
-50	-0.69	-0.63
-40	-0.51	-0.48
-30	-0.36	-0.35
-20	-0.22	-0.22
-10	-0.11	-0.11
+00	0.00	0.00
+10	0.10	0.10
+20	0.18	0.18
+30	0.26	0.26
+40	0.34	0.32
+50	0.41	0.38

Source: Markowitz (1959), p. 121, table 2 or Markowitz and Blay (2013), pg. 40, table 2.1.

Harry always referenced the table on page 121 of his 1959 book (see table 1 with some useful additional headers and shading). This is also included on page 40 of Volume 1.

As table 1 shows, the mean-variance approximation does a good job for returns between -30% and +50% (the shaded area), and not too bad of a job outside of that. Since the days when this table was initially produced, research on mean-variance approximations to expected utility has advanced. Harry details how there is now more than fifty years of extensive, research showing that certain functions of mean and variance do quite a good job of approximating various utility functions for a variety of return distributions. Harry closes Chapter II addressing "The Great Confusion" with the following: "It is now over a half-century since Markowitz (1959) first defended MV [mean-variance] analysis as a practical way to approximately maximize EU [expected utility]. In light of repeated confirmation of the efficacy of MV approximations to EU, the persistence of the Great Confusion – that MV analysis is applicable in practice only when return distributions are Gaussian or utility functions quadratic – is as if geography textbooks of 1550 still described the world was flat."

The remainder of Volume I explores mean-variance approximations to geometric mean, or rather, long-run returns, compares mean-variance approximations with other measures of risk that supposedly better account for non-normal distributions, and explores what types of return distributions investors are most likely to encounter joint work. All of these analyses served to bolster the argument in favor of mean-variance and to further punctuate his response to "The Great Confusion."

In Volume II,[16] Harry focuses on the context of portfolio selection and discusses portfolio analysis over multiple periods. Among other things, he discusses how to model dynamic systems, where his work on SIMSCRIPT and simulation are clearly evident and where he makes the case for the importance of simulation and decision support systems (DSS). He delves into game theory and the limitations of dynamic programming, and shares an unfinished argument with Nobel laureate Paul Samuelson about return in the long run. He then shares a variety of different advances in portfolio selection, including our joint work on tax-cognizant portfolio analysis, and discusses the importance of judgement and approximation. He finishes the volume with a chapter titled The Future, where he provides detailed guidance on developing the financial simulators and decision support systems of the future. I am hopeful that someone will follow his lead on developing what he laid out.

In Volume III,[17] Harry discusses rational decision making under uncertainty, or rather, when the odds are not known. This expands on Chapter 12 of Markowitz (1959), titled Probability Beliefs, and focuses on how to go from information to action. This is arguably the most historical and deeply philosophical volume, as it discusses Rene Descartes, David Hume and others, and presents first principals of deduction and induction theory, among other things. What was originally presented in 17 pages now spans a full 296+ page volume and provides the philosophical and theoretical lineage of Harry's beliefs.

Sadly, Harry completed only three of the four planned volumes of the book. However, true to his passion for learning, he had continued to advance work on portfolio theory exclusive of his work on the book.

Harry Markowitz, the man

Working through the portfolio selection process step-by-step and advancing asset allocation research with Harry was an extraordinary life experience. But the greatest privilege in working with Harry was getting to know him as a person and a friend.

Invesco and Markowitz

In 2018, Harry accepted my invitation to work together with Invesco's Investment Solutions team to support ongoing research on multi-period portfolio selection. This would advance previous research we had done developing an optimization framework that considered the impact of taxes on wealth outcomes (Blay and Markowitz, 2016). That work required a multi-period perspective given the path-dependent nature of taxation.

The research we developed with Harry at Invesco substantially advanced the state of multi-period portfolio theory. Current multi-period portfolio selection methods are largely based on dynamic programming approaches first suggested by Markowitz (1959) and then progressed by Mossin (1968), Samuelson (1969) and Merton (1969). Unfortunately, these approaches suffer from what Richard Bellman termed 'the curse of dimensionality', which states that the computing power required to solve dynamic programming problems increases exponentially as the number of state variables increases. Solving practical multi-period portfolio selection problems with a standard set of state variables is still beyond the computing power available today.

Our approach, which we now call Simulation-Based Portfolio Selection (SBPS), leverages simulation to not only overcome computing power limitations but also address what we have determined to be three requisites for practical multi-period solutions: (1) They must evolve allocations and duration over time to align with expected cash flows, (2) They must consider real-world asset dynamics, and (3) They must consider investment frictions and illiquidities.

We were incredibly fortunate to have benefitted from Harry's expertise in both portfolio selection and market simulation in developing SBPS. It marks a great leap forward in practical multi-period portfolio selection and was Harry's final extension of portfolio theory.

Harry loved people. He loved talking about ideas.

Harry was a joyful and generous person. We had regular "one-hour" calls twice a week, where we often talked for two or three hours, or more. There was a lot of joking, laughing, and even singing of old show tunes (mostly, if not exclusively, by Harry). Somehow, with all of that we would still manage to talk about portfolio analysis and the research at hand. He would patiently walk me through how we should do things and, most importantly, why we should do them in a particular way. He was always open to my questions, ideas, and even my challenges and was invariably practical and constructive.

As we worked through problems, we would inevitably get to a point where we needed to develop something before we could proceed. This would become my assignment. For example, he'd announce, "we need a model of covariance," or "we need a taxation algorithm," or "we need a taxation simulator." I would then go to work on the assigned task. I would often send him my results in the afternoon and get a response in the wee hours (1:00 AM or 2:00 AM in Harry's time zone) the following day for discussion on our next call.

What I experienced – and what I've often heard about Harry from other researchers, and even his own family – was that they couldn't understand how he could give them so much of his time, laughing and joking, walking to and from lunch when visiting San Diego, discussing ideas, and somehow get so much done. My answer is that Harry loved people. He loved talking about ideas, reminiscing about what he'd done and dreaming about what was still left to do (even in his 90's). Being with people energized him – and so he would work late into the night because, as he explained, it was quiet and he could focus.

Unfortunately, the COVID pandemic took its toll on Harry. He never contracted the virus, but his advanced age meant that he was isolated from social interactions. It wasn't until then that I sensed my friend had grown old.

I had dinner with Harry at his home a few months before he passed. I had the feeling it might be the last time we would share time together. So, I thanked him for everything he had done for me – for his patience in teaching me, his guidance and mentorship, and for just being a great friend. He was as gracious as always. I then asked him what his most important accomplishment was in life. Without hesitation, he pointed to a set of photos that hung on the dining room wall. They were all pictures of his family, from his wife to his many great-grandchildren.

As we finished dinner, I repeated an old George Burns joke I often told when he mentioned his age: "Harry, if you live to be one hundred, you've got it made. Very few people die past that age." Harry laughed. He always liked that joke. Unfortunately, Harry didn't make it to one hundred. However, his ideas will be with us for well beyond that. And, at least in that sense, it's comforting to know that Harry does have it made.

Notes

1 Keynes (1942, 1983).
2 Bernstein (2002).
3 Leavens (1945).
4 Markowitz (1952a).
5 Bernstein (2007).
6 Markowitz (1952b).
7 Goodman and Markowitz (1952).
8 Savage (2009).
9 Dimand (2009).
10 Markowitz (1959).
11 Sharpe (1963).
12 Sharpe (1964).
13 Markowitz (2005), Markowitz (2008).
14 Markowitz (1991).
15 Markowitz and Blay (2012).
16 Markowitz (2016).
17 Markowitz (2020).

About the author

Kenneth Blay
Head of Research
Global Thought Leadership
In this role, he leads the development of original research through collaborations with investment teams, industry researchers and institutional clients.

Additional notes

Page 11: Markowitz was willing to challenge his own professors

In his Utility of Wealth paper, Harry argued against Milton Friedman and Leonard J. Savage on work they had done posing a solution to why people simultaneously purchase insurance and lottery tickets, a classic question in understanding how people behave. Both Friedman and Savage had been Markowitz's professors.

Page 11: Savage, statistics, and Markowitz

The ideas Savage shared with Harry were unquestionably revolutionary. In 1954, Savage published Foundations of Statistics which put forward a theory of subjective and personal probability. This challenged the then dominant frequentist school of statistics and initiated one of the greatest controversies in modern statistical thinking. Harry would often say that he learned statistics at point blank range from Leonard J. Savage. He would also proudly proclaim that he was a Bayesian.

Page 11: An ultra-brief summary of expected utility theory

Expected utility theory is a framework for decision making under uncertainty where decision makers ascribe a value to potential outcomes based on specified preferences. The higher the value, or expected utility, the more desirable the outcome. Utility functions like ln(1 + return) are often used to describe specific preferences.

Page 12: Portfolio theory, mathematics and you

Harry's idea for the book was to explain portfolio theory to someone with no mathematical training. That didn't mean math wasn't involved. It simply meant that he would teach the reader the required math. In fact, Chapter III begins with a section titled Mathematics and You. I learned matrix algebra from Chapter VIII of his book… something Harry shared with me that Bill Sharpe did as well.

Page 12: Required reading for asset allocation practitioners

Along with reading Harry's 1959 book, practitioners would also be well served by reading Risk-Return Analysis: The Theory and Practice of Rational Investing Volumes I-III, where he revisits Part IV of his 1959 book and provides his final thoughts on the subject.

Page 12; Markowitz, simulation and models.

Harry was drawn to simulation partially because he understood the limits of models. Even though the CAPM was based on his ideas, he published papers highlighting some shortcomings of the model. Two of these papers were Market Efficiency: A Theoretical Distinction and So What? and CAPM Investors Do Not Get Paid for Bearing Risk: A Linear Relationship Does Not Imply Payment for Risk. Harry shared with me that, out of respect, he reached out to Sharpe to discuss the implications of this work before publishing.

Page 13: The origins of the CAPM

While the CAPM is generally attributed to Sharpe (1964), it is also recognized that many others, including Treynor (1962), Lintner (1965) and Mossin (1966), contributed to the concept we understand today. For example, Jack Treynor had shared mimeographed copies of his ideas about a capital asset pricing model in academic circles before Sharpe's article. However, he never published his work.

Page 15: Markowitz versus Samuelson

Markowitz and Samuelson had an ongoing and contentious debate about how investors should invest over the long run. As part of this argument Samuelson published a paper making his point using only one-syllable words except for the last one. His point was to make his argument easier to understand. Harry replied saying, "It is hard not to feel intimidated in a debate with an opponent who is a combination of Albert Einstein and Dr. Seuss." Unfortunately, Samuelson passed away before they could arrive at a mutual resolution. Ultimately, Harry showed how the mathematical facts supporting both arguments can both be true. However, he also points out that his argument aligns with the more common understanding of what is meant by the "long run".

Page 15: Markowitz's original thinking on multi-period solutions

In his 1959 book, Markowitz provided guidance on the problem of portfolio selection through time. He explained how, under certain conditions, a dynamic programming approach could be used to provide an exact solution. He conceded, however, that dynamic programming techniques were probably infeasible due to the computational requirements of even the simplest utility functions. In his final work on multi-period portfolio selection, completed in collaboration with Invesco, we were able to provide a highly flexible solution that avoids the curse of dimensionality faced by dynamic programming approaches.

Page 16: The importance of family

Aside from the comments Harry shared with me about family the last time I saw him, I recall when Risk-Return Analysis: The Theory and Practice of Rational Investing, Volume I was first published. It was hard to tell if he was more excited about the fact that the dedication was so long with him listing all of his family (children, grandchildren, and great grandchildren) or if it was about the book being published generally.

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