2020 Long-Term Capital Market Assumptions

Methodology

Invesco Investment Solutions | United States Dollar (USD)

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"As explained in the first paragraph of my (1952) paper, ideally security analysts [should] make forecasts about security returns' means, variances and covariances. Portfolio analysts should then use these estimates to compute and present portfolio opportunities."

Harry Markowitz, Economist, Nobel Laureate

Harry Markowitz
Economist
Father of Modern Portfolio Theory
Invesco Investment Solutions
Research Partner

Foreword

A saying, whose source is lost in antiquity, is that "In theory, there is no difference between theory and practice; in practice there is." This is true, for example, for the "Theory of The Firm." No one runs a firm using the Theory of the Firm. They couldn't. The economist's theory of the competitive firm has a continuous production function. The firm chooses the production level that equates marginal cost to marginal revenue. In practice, to increase production with given workers and equipment involves working overtime with overtime pay. To acquire a new plant, equipment, or even appropriately skilled workers takes time and resources. Clearly, for a manufacturing firm, the Theory of the Firm is an enormous abstraction from the real firm.

On the other hand, portfolios are, in fact, chosen with the help of portfolio theory. As explained in the first paragraph of my (1952) paper, ideally, security analysts [should] make forecasts about security return's means, variances and covariances. Portfolio analysts should then use these estimates to compute and present portfolio opportunities, and the client should choose that which they want to be implemented.

Differences between theory and practice also exist when it comes to portfolio implementation. Paul Samuelson and I had differing views on "Investment for the Long Run." I accepted the Kelly (1956) Maximum Expected Logarithm—or MEL—rule for investing in the indefinitely long run; Samuelson did not. We debated this in the literature. See Markowitz (2016) for my final words on the subject and Samuelson (1979) for his. We also debated face to face, such as at a "Q-Group" meeting attended by the top technical people, as compared to top administrative people, of large institutional and corporate investors. One point that Samuelson made at this meeting was that portfolios are actually selected with the help of portfolio theory. This is as compared to the economist's "Theory of the Firm", which is not specific enough to run a firm.

Here one must distinguish between the theoretical investor postulated by Mossin (1968) and Samuelson (1969), and an actual individual or institutional investor. In particular, the MS investor deposits funds into an account, then neither adds nor withdraws from the account until his or her retirement. He or she can change its stock/bond ratio. The paradoxical Mossin-Samuelson result is that the investor never changes its asset mix even when retirement is imminent.

Since that behavior is neither seen nor plausible, one must seek investment/consumption rules that make sense. In 1952, when the modern computer was in its infancy, the above-mentioned rules were a general guide to consumption/investment action. Now it is implemented in a program that helps implement one's action. The question is what these rules should be, and how should the individual — the human — help guide-*and use*- its advice **for real**.

This paper by Invesco's Investment Solutions team presents their work in developing forecasts for asset class returns, risk, and correlations, which are of critical importance in translating portfolio theory into plausible real-world practical solutions for investors.

Sincerely,

References:

Kelly, J.L., Jr. 1956 "A New Interpretation of Information Rate." *Bell System Technical Journal* 35: 917-926.

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Mossin, J. 1968. "Optimal Multiperiod Portfolio Policies." *Journal of Business* 41(2): 215-229.

Samuelson, P.A. 1969. "Lifetime Portfolio Selection by Dynamic Stochastic Programming." *Review of Economics and Statistics* 51(3).

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01

Methodology Overview

Returns in the methodology are presented in USD and are geometrically linked, but are developed arithmetically and in most currencies.

Estimating returns for asset classes: A building block approach

We employ a fundamentally based "building block" approach to estimating asset class returns. Building blocks represent a "bottom-up" approach in which the underlying drivers of asset class returns are used to form estimates (**Figure 1**).

First, these sources of return are identified by deconstructing returns into income and capital gain components. Next, estimates for each driver are formed using fundamental data such as yield, earnings growth and valuation, and combined to establish estimated returns.

By incorporating fundamental data, our approach allows for the relative attractiveness of asset classes to vary over time. Other approaches based on historical relative returns can provide relatively static risk-premiums through time in which certain asset classes contain constant return advantages. The following sections will detail and present the estimates across various equity, fixed income and alternative asset classes.

Figure 1: Our building block approach to estimating returns

■ Income ■ Capital gain ■ Loss

	Equity	Fixed income	Direct real estate
Expected returns	Total yield	Total yield	Income
	+ Valuation change	+ Valuation change	+ Valuation change
	+ Earnings growth	+ Roll return	+ Growth
		- Credit loss	

For illustrative purposes only.

02

Equities

To reflect the impact of both dividend yield and buybacks, we base the estimate for total yield on the 10-year average total yield ratio.

The building block methodology reflects a total return approach to equities – accounting for both income and capital appreciation (i.e., the change in price over time). The building blocks, therefore, consist of estimates for yield (as a driver of income) and earnings growth and valuation change (as drivers of capital appreciation). We begin by looking at large-cap US equities.

Total yield

■ Income ■ Capital gain

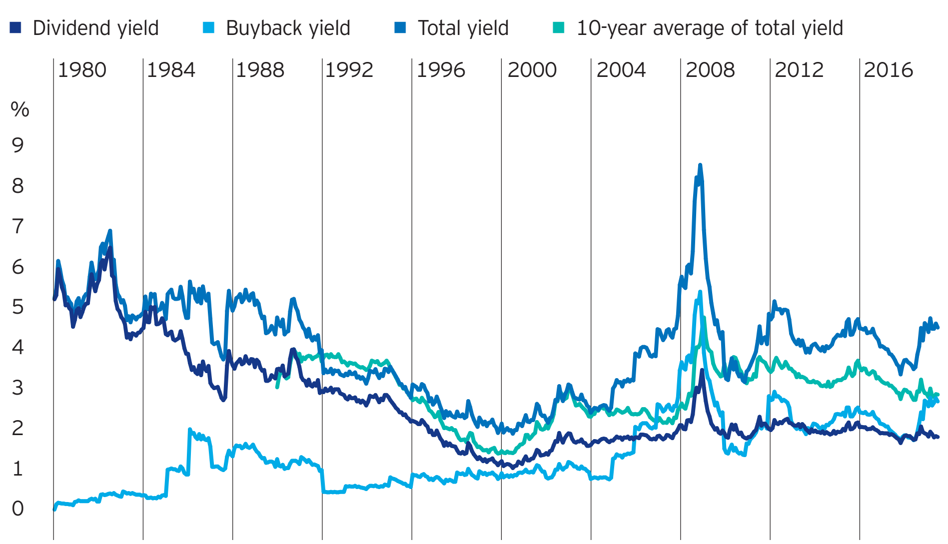
	Large-cap equities
Expected returns	**Total yield**
	+ Earnings growth
	+ Valuation change

For illustrative purposes only.

Our approach to estimating yield is based on the 10-year average total yield ratio, which reflects the impact of both dividends paid and shares repurchased by the firm. Estimating the former is relatively straightforward, using current dividend yield – dividend per share divided by the price. Repurchased shares, also known as buybacks, involve a company purchasing some of its outstanding shares, thereby reducing the number available on the open market. We believe it's important to capture the impact of buybacks, particularly in the US, given the structural changes in the US tax code dating back to the 1990s. These changes resulted in a dramatic increase in share buybacks in place of dividends over the past 20 years, which benefited returns in the form of capital gains over income. While buybacks themselves do not generate income, they represent an alternative way for firms to return capital to shareholders. Given the dramatic decrease in payout ratio due to buyback activity, we account for the effect of buybacks in our yield calculation to provide more meaningful return estimates. We estimate using the 10-year average of the total yield ratio to bridge the gap in terms of how capital is transferred (**Figure 2**).

Total yield = Dividend yield + Buyback yield

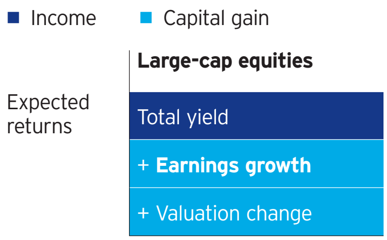
Figure 2: We apply the 10-year average real total payout to current real price to proxy total yield



Source: FactSet Research Systems Inc. from Jan. 31, 1980 to Sept. 30, 2019. Based on S&P 500 Index.
Past performance does not guarantee future results.

Real GDP per capita provides a stable signal over time to estimate earnings growth.

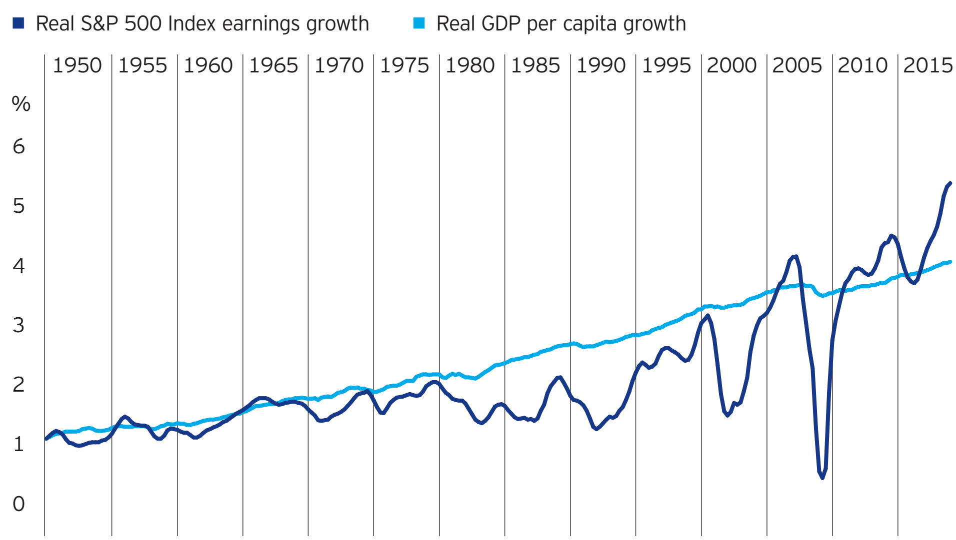
Earnings growth



For illustrative purposes only.

Growth of earnings per share is one of two significant drivers of capital appreciation in stock returns. Although past earnings could provide important insight into estimating the growth of future earnings, this approach is not well-suited due to the volatility in earnings levels that arises from market fluctuations and accounting charges. Given our longer-term outlook, we prefer a more stable estimate of earnings growth through time. Historically, there has been a strong relationship between real US gross domestic product (GDP) per capita growth and real S&P 500 Index earnings growth (**Figure 3**). Consequently, we use real GDP per capita – which also appears to have been a more stable signal over time – to estimate earnings growth in the model and apply future inflation expectations to that estimate to forecast nominal earnings growth. We use the long-term average because we believe that in the case of developed economies, they are less likely to deviate significantly from their "steady state" growth levels.

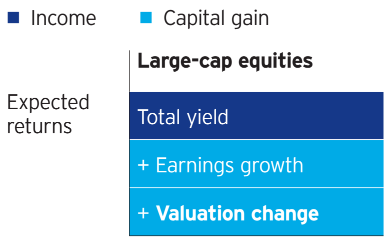
Figure 3: Over the long run, real S&P 500 Index earnings growth has tracked real US GDP per capita growth



Sources: Robert Shiller Yale Data; FactSet Research Systems Inc. and St. Louis Federal Reserve from Jan. 31, 1950 to March 31, 2019. Latest data available as of Sept. 30, 2019.

The first step to estimating valuation change is calculating a long-term mean for the P/E ratio.

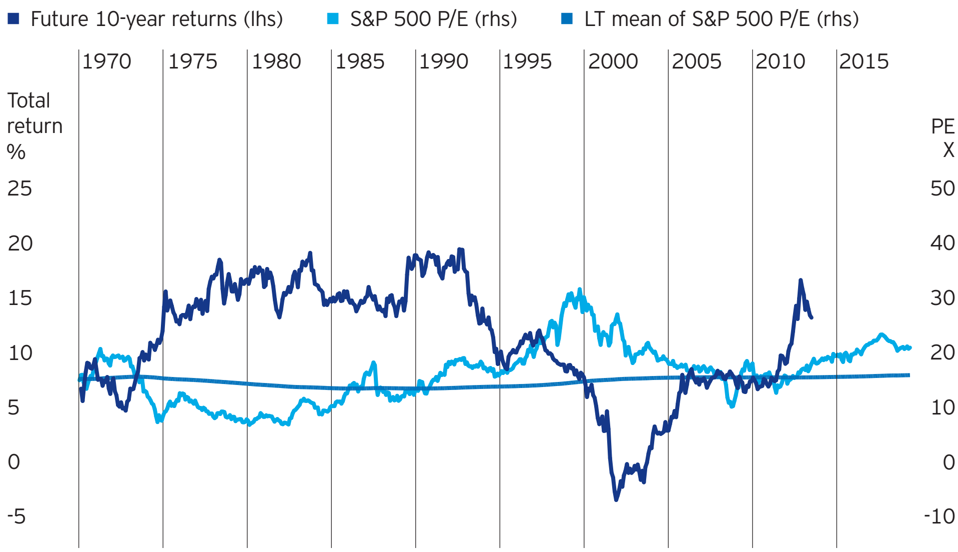
Valuation change



For illustrative purposes only.

The second significant driver of capital appreciation in stock returns is the change in equity valuation – in terms of the ratio of price to earnings (P/E) – over time. In estimating P/E, we recognize existing research (Campbell and Shiller, 1998), which suggests that over time, the P/E ratio should revert to its long-term mean. In other words, if equities are currently considered "cheap," which means that the current P/E is lower than the long-term average, there should be a catalyst to revert the P/E back to the mean (**Figure 4**).

Figure 4: The P/E ratio of the S&P 500 Index has tended to revert to the mean



Sources: Robert Shiller Yale Data; FactSet Research Systems Inc. from Feb. 28, 1970 to Sept. 30, 2019. These estimates are forward-looking, are not guarantees, and they involve risks, uncertainties, and assumptions. These estimates reflect the views of Invesco Investment Solutions, the views of other investment teams at Invesco may differ from those presented here.

Therefore, our first step in estimating the change in equity valuation is to calculate a long-term mean of the P/E ratio. Consistent with academic literature (Lee, Myers and Swaminathan, 1999), we found that the long-term mean of the P/E ratio is a function of prevailing macroeconomic conditions, including the interest rate environment and inflation, as these affect how much an investor would be willing to pay for equities. We model the mean of the long-term P/E ratio through regression analysis, using monthly data.

Figure 5: Estimating the long-term mean of the P/E ratio using regression analysis

1. A regression of monthly data (January 1970–September 2019) yielded the following coefficients:

   $P/E = a + bRF + c\,\pi$

2. To determine the long-term mean of the P/E ratio, we use the results of the regression analysis along with the figures for the risk-free rate and inflation, which as of Sept. 30, 2019, totaled 1.70% and 1.76%, respectively:

   $P/E = 20.80 + (-0.52 \times 1.70) + (-0.59 \times 1.76) = 18.87$

3. Looking at this empirical data, we found that P/E is negatively related to the risk-free rate and inflation because investors require higher returns as they increase.

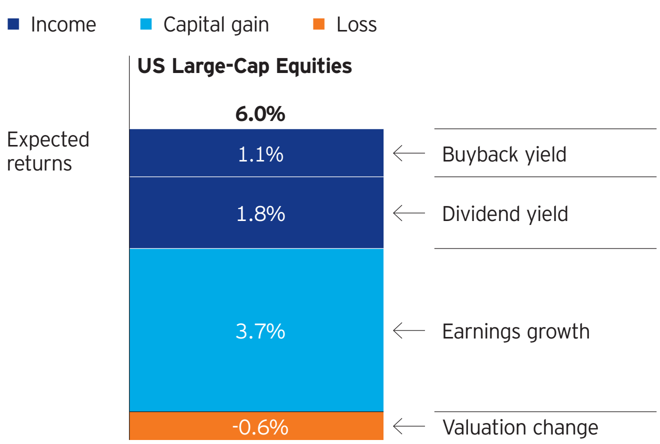
4. Next, to estimate the potential for valuation change, we look at current valuation relative to a rolling average P/E, as estimated in the above regression analysis. The change in valuation is then annualized, or amortized, over the 10-year time horizon, so that it can be either added to or discounted from the total return estimate:

$$\text{Valuation change} = \left(\frac{\widehat{P/E}}{P/E\ Current}\right)^{1/10} - 1$$

Source: Federal Reserve Bank of St. Louis. As of Sept. 30, 2019. This is over a five-year rolling period based on the S&P 500 Index. RF = Risk free rate; π = Inflation; a = 20.80; b = -0.52; c = -0.60

We then include a scaling factor to account for dislocation in valuation. In other words, extreme dislocations in P/E (high or low versus the average) have a larger impact on estimated returns.

Figure 6: Putting it all together: Building blocks of US Large-Cap Equities



Source: Invesco as at Sept. 30, 2019. US Large-Cap Equity is represented by the S&P 500 Index. These estimates are forward-looking, are not guarantees, and they involve risks, uncertainties, and assumptions. These estimates reflect the views of Invesco Investment Solutions, the views of other investment teams at Invesco may differ from those presented here.

Beyond US large cap: Consistent approach across all equity classes

In terms of estimating returns for other equity sub-asset classes, one of the benefits of the building block approach is that it's very "portable" – meaning, it can be applied uniformly across all segments of equities including size (mid and small cap), style (growth/value), and geography (non-US developed, emerging markets).

Let's take a closer look at some examples:

+ **US small-cap equities.** US small-cap equities share the same drivers of return as large-cap equities – yield, earnings growth and valuation change. We estimate return for small-cap equities by looking at those drivers in the context of the US small-cap benchmark, the Russell 2000 Index.
+ **Non-US equities.** Our research indicates that, with the exception of the impact of share repurchases on estimating yield (as previously discussed), non-US equities share the same drivers of return as US equities, but are evaluated in the context of the representative benchmark (e.g., MSCI EAFE Index, MSCI World Index).

Figure 7 highlights our approach for estimating returns for the various segments of the market.

Figure 7: Applying building block methodology to equity sub-asset classes

■ Value change ■ Earnings growth

	Large-cap equity	Small- to mid-cap equity	Non-US equity
Expected returns	Mean P/E reversal of each index x scaling factors	Mean P/E reversal of each index x scaling factors	Mean P/E reversal of each index x scaling factors
	+ Long-term real US GDP per capita growth + Expected US inflation	+ Earnings growth for the different market segments are scaled relative to their respective large cap index	+ Long-term real GDP per capita growth of each country + Regional inflation expectation
	+ Dividends + buybacks = Total yield	+ Dividends + buybacks = Total yield	+ Dividends + buybacks = Total yield

Source: Invesco. For illustrative purposes only.

Figure 8: 10-year estimated equity market total returns (USD)

Asset class	Index	Estimated return %	Yield %	Earnings growth %	Valuation change %	Currency adjustment %
Emerging market	MSCI EM	8.63	2.94	4.96	0.73	0.00
Developed ex-US	MSCI World Ex-US	6.29	3.33	1.35	-0.03	1.64
US large-cap	S&P 500	6.02	2.88	3.70	-0.57	0.00

Source: Invesco, estimates as of Sept. 30, 2019. All total returns data is annual. For illustrative purposes only. These estimates are based on our capital market assumptions which are forward-looking, are not guarantees, and they involve risks, uncertainties and assumptions.

Developed markets versus emerging markets equity CMAs

Emerging market (EM) economies are structurally different than developed markets (DM), leading to differences in the way their CMA building blocks are estimated. Even amongst the broad EM category are economies different enough to justify individual marginal adjustments. Some EM economies, like Korea and Taiwan, are more mature in their economic development, are export-oriented, and have similar characteristics to DM economies. China, however, is a high growth economy, driven by credit and money growth, that is moving to more long-term sustainable growth levels and is much more oriented to domestic growth.

Emerging Market Building Blocks (including Hong Kong, as more than half of market capitalization of Hong Kong-listed firms are Chinese companies) :

Emerging Market Building Blocks:
+ **Yield.** For Korea and Taiwan, our yield approach is the same as DM (ex-US). Hong Kong's market focuses on recurring trailing dividend yield. With many Hong Kong-listed firms being family or state-owned, non-recurring special dividends can occur, so we exclude special dividends from our analysis to prevent outliers within our yield estimates.
+ **Earnings growth.** For Taiwan and Korea, we follow the same process as for DM. China used to be a high growth market and is now slowing down. Because of this, we adjust the historical average growth by calculating the decline of other Asian economies that have been in similar economic positions and deduct that figure from China's 10-year average real GDP growth.
+ **Valuation change.** Similar to DM, we assume valuation such as the price-to-book ratio will return to the long-term mean after adjusting for macroeconomic variables. For Taiwan and Korea, exports make up more than 60% and 40% of GDP, respectively. As export-driven economies, currency (FX) has a bigger impact than inflation on valuation change. For Hong Kong, growth is influenced by inflation in China. As the HK$ is pegged to the USD, we do not look at FX but look at the HK "risk-free" rate as liquidity conditions are influenced by the HK$ peg and, therefore, the US Fed's monetary policy.

Figure 9: Relative valuation adjustments of EM economies based on their economic characteristics

Classification	Inflation	Rates	FX	TSF growth	Applies to
Developed markets	✓	✓			Regional CMAs Hong Kong (USD peg)
Export oriented mature emerging economies		✓	✓		Taiwan, Korea
Domestic oriented emerging economies	✓			✓	China

Source: Invesco, as of Sept. 30, 2019.

To test the accuracy of our CMA's we review the realized versus predicted returns of US large cap, developed ex-US, and emerging markets. All possible estimate history available is presented.

Any asset class' accuracy chart can be provided upon request. Please reach out to the IIS Global Client Solutions contact on the last page of the document.

Figure 10: US large cap: CMA returns vs actual returns (S&P 500 Index)

Source: Invesco. Data from Jan. 31, 1973-Sept. 30, 2019. An investment cannot be made directly into an index. Capital market assumptions are forward-looking, are not guarantees and they involve risks, uncertainties and assumptions. These estimates reflect the views of Invesco Investment Solutions, the views of other investment teams at Invesco may differ from those presented here.

Figure 11: DM ex-US: CMA returns vs actual returns (MSCI World ex-US Index)

Source: Invesco. Data from Jan. 31, 2000-Sept. 30, 2019. An investment cannot be made directly into an index. Capital market assumptions are forward-looking, are not guarantees and they involve risks, uncertainties and assumptions. These estimates reflect the views of Invesco Investment Solutions, the views of other investment teams at Invesco may differ from those presented here.

Figure 12: EM: CMA returns vs actual returns (MSCI EM Index)

Source: Invesco. Data from Jan. 31, 2001-Sept. 30, 2019. An investment cannot be made directly into an index. Capital market assumptions are forward-looking, are not guarantees and they involve risks, uncertainties and assumptions. These estimates reflect the views of Invesco Investment Solutions, the views of other investment teams at Invesco may differ from those presented here.

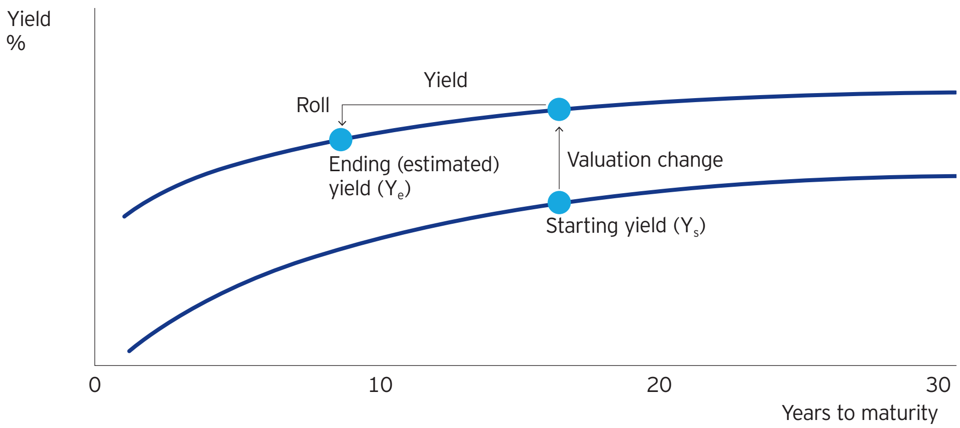
03

Fixed Income

The estimate for total yield reflects the impact on income from changes of the yield curve over time.

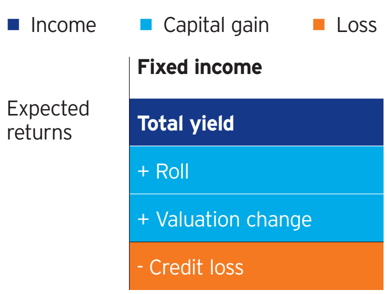
Within fixed income, we also utilize the building block methodology, seeking to isolate and identify the individual drivers of the specific asset class risk premium. As with equities, the drivers of return for fixed income are income (yield) and appreciation (roll return, valuation change, and credit loss).

Figure 13: Single-period return decomposition



Source: Invesco. For illustrative purposes only.

Total yield



For illustrative purposes only.

Yield reflects the average income expected to be received from an investment in a fixed income security throughout its life. For the purposes of our CMAs, yield is calculated using an average of the starting (current) and ending (estimated) yield levels.

To calculate the ending (estimated) yield (Y_e), we examine how the current (starting) yield curve (Y_s) could move over time as a result of changes in Treasury interest rates and in the credit spreads over US Treasury interest rates.

$$Y_e = Y_s + \Delta Y_{TSY} + \Delta OAS$$

ΔYTSY = Changes in Treasury interest rates (at a given duration); *ΔOAS* = Changes in credit spreads over US Treasuries

For non-US assets, we use the yield curve estimates for that region.

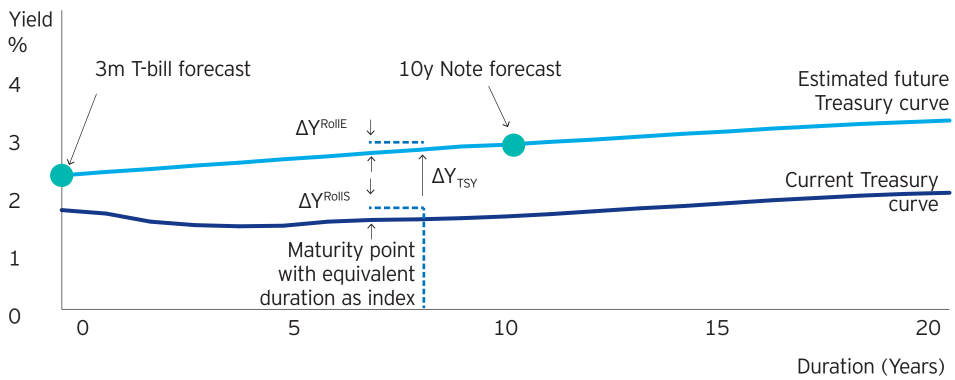
Changes in Treasury interest rates *ΔYTSY*
As suggested in the relevant academic research (Litterman and Scheinkman, 1991), changes in Treasury interest rates have the potential to affect the position and shape of the future Treasury yield curve, in terms of its level and slope relative to the starting (current) yield curve.

Charting the future Treasury yield curve involves:
+ Identifying the yield for three-month Treasury bills and the yield for 10-year Treasury notes, as two specific points which help determine the level (intercept) and slope (**Figure 14**).
+ Polynomial interpolations is then applied using these two data points, which are sourced from the consensus forecasts of the Federal Reserve Bank of Philadelphia, to generate the estimated future yield curve.
+ For the purposes of estimating the impact of changes in Treasury interest rates on estimated yield *ΔYTSY,* we take the difference in yields at a specific duration between the current and estimated future yield curves.

$$\Delta Y_{TSY} = i_{estimated} - i_{current}$$

ΔYTSY = Changes in Treasury interest rates (at a given duration); ΔOAS = Changes in credit spreads over US Treasuries

Figure 14: Treasury curve estimate based on Federal Reserve Bank of Philadelphia consensus



Source: Invesco. For illustrative purposes only.

Changes in credit spreads *ΔOAS*
Another factor impacting the direction of estimated future yield involves movement in credit spreads, which historically have exhibited mean-reverting properties (Prigent et al., 2001). This means, for example, that if spreads are currently very wide relative to the mean, our forward expectations are for spreads to narrow, and for that contraction to have a positive impact on pricing.

We estimate the changes in that spread by looking at the relationship between current credit spreads and their 10-year rolling average (**Figure 15**). We cap the potential movement in credit spreads to 10% in order to mitigate the impact of extreme credit events (e.g., the global financial crisis).

$$\Delta YOS = OAS_{current} - OAS_{10\text{-year average}}$$

ΔYTSY = Changes in Treasury interest rates (at a given duration); *ΔOAS* = Changes in credit spreads over US Treasuries

Figure 15: High-yield credit spreads revert to the long-term (10-year) average

Source: FactSet Research Systems Inc. from Jan. 31, 2003 to Sept. 30, 2019. Option-adjusted spreads (OAS) account for bonds with embedded options, such as callable bonds.

Figure 16: Estimating total yield

Maturity = 6 years
Starting yield = 2.26%

Estimated yield
Movement in interest rates
- *Interest rates at a maturity of six years on the current yield curve = 1.62%*
- *Interest rates at a maturity of six years on the future yield curve = 2.75%*

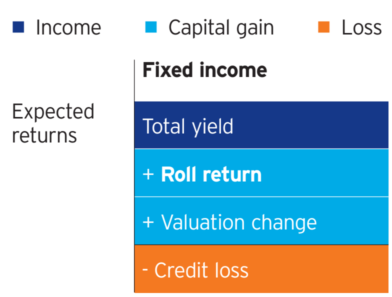
Movement in credit spreads
- *Current credit spread = 0.46%*
- *Rolling 10-year credit spread = 0.62%*

Ending yield = 2.26% + (2.75% - 1.62%) + (0.62% - 0.46%) = 3.55%

Yield estimate = (2.26% + 3.55%)/2 = 2.91%

Source: Invesco Investment Solutions Research. Data as of Sept. 30, 2019.

Roll return



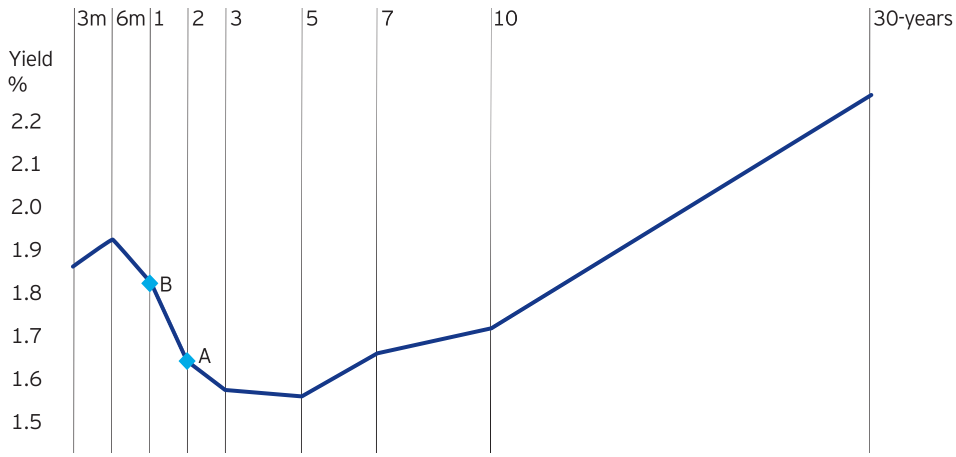
For illustrative purposes only.

Roll return reflects the impact of movement along the curve – over the passage of time – on the potential return of a fixed income security (i.e., appreciation). Specifically, it looks at the impact on price, all else being equal (i.e., no movement of the yield curve), as a bond nears maturity. If the yield curve slopes upward, movement along the curve (toward maturity) will make a positive impact on returns.

Roll return reflects movement along the yield curve – the impact on price from holding a bond over time.

Let's take a closer look at how this works (**Figure 17**). Consider the current upward sloping yield curve of "on-the-run" (i.e., the most recently issued) US Treasuries with maturities extending from zero to 30 years. Assume that we purchase a two-year US Treasury note (point A), which yielded 1.61% on Sept. 30, 2019. Assuming no changes to the yield curve, a year from now, the maturity of the note would have decreased to one year, which corresponds to a yield (on the current yield curve, which has not changed/moved) of 1.76% (point B). Given the inverse relationship between the price and yield on bonds, in order for the yield on the note we purchased to increase, the price of the note needs to decrease – which represents the capital loss.

Figure 17: Roll return reflects the impact on yield and price as a bond is held over time



Source: Bloomberg L.P., as of Sept. 30, 2019. At Maturity = t, the roll return is calculated as follows: Roll return = -(t - 1) xΔy; Δy = Interest rate$_{t-1}$ - Interest rate$_t$

Figure 18: Estimating roll return

In order to determine the roll return, for methodological simplicity, we choose to focus only on the roll impact along the Treasury curve. Similar to the yield computation, we again rely on the average of the starting and estimated future roll and compute the roll return as follows.

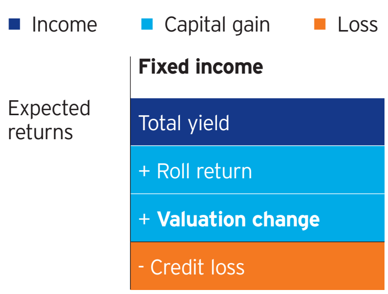
Interest rate on current yield curve at:
6-year maturity = 1.62%
5-year maturity = 1.55%

Interest rate on future yield curve at:
6-year maturity = 2.75%
5-year maturity = 2.70%

Current roll return = -5 x (1.55% - 1.62%) = 0.34%
Future roll return = -5 x (2.70% - 2.75%) = 0.28%
Roll return = (0.34% + 0.28%)/2 = 0.31%

Source: Invesco. Data as of Sept. 30, 2019.

Valuation change



For illustrative purposes only.

Valuation change reflects the impact on price from movement of the yield curve.

If roll return incorporates the impact on the price of movements along the curve, valuation change reflects the impact on price from movement of the curve. Another way to think about valuation change is that it examines the same dynamic we explored in defining the building block to estimate the return from yield (see **Figure 19**) but looks at this movement's impact on price, rather than income. As discussed above in the context of returns from yield, this comprises movement due to changes in interest rates and credit spreads, respectively.

Figure 19: We estimate the impact of this change as follows:

For Maturity = t

Valuation change $= 1 - [1 - t \times (Y_e - Y_s)]^{1/10} - 1$

From the total yield calculation we know that:

$Y_e - Y_s = \Delta Y_{TSY} + \Delta OAS$

In other words, the change in yield reflects changes in duration and credit spreads:

Valuation change $= [1 - t \times (\Delta Y_{TSY} + \Delta OAS)]^{1/10} - 1$

Figure 20: Estimating valuation change

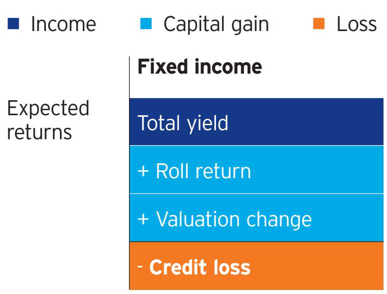
Maturity = 6 years
Current yield = 2.26%
Ending yield = 3.55%

Valuation change = [1 - 6 x (2.26% - 3.55%)] 1/10 - 1 = -0.80%

Source: Invesco Investment Solutions Research. Data as of Sept. 30, 2019.

Credit loss

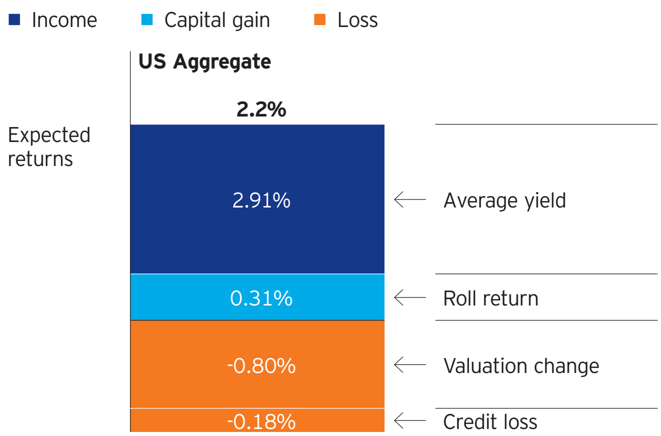
The estimated impact on return from:
+ Bond migration = option-adjusted spread x 40% "haircut"
+ Estimated default loss = 10-year median of annual default rates x Average 40% recovery rate



For illustrative purposes only.

Credit loss captures the potential impact on returns from a downgrade in credit ratings (i.e., bond migration) and from a debt default. Let's examine each of these potential sources of loss:
+ **Bond migration.** For investment-grade bonds, downgrades – particularly those that place a security below investment grade level – could have a negative impact on returns, as these bonds entail a higher yield, which would drive down prices. The estimated impact on return from this process can be estimated by multiplying the option-adjusted spread (OAS) – which measures the spread between a fixed income security and the risk-free rate of return, which is adjusted to account for an embedded option – by the "haircut," a reduction in the stated value of an asset. Our rationale for this methodology is based on observations of historical data, which indicate that loss from credit migration increases as the OAS widens. Also, based on historical data, we use a static 40% as the haircut estimate.
+ **Estimated default loss.** For riskier fixed income instruments such as high yield, floating rate, preferred stocks and emerging market bonds, default is a more significant driver of potential credit loss. The estimated default loss is a function of the estimated default rate, which is based on the 10-year median of annual default rates published by Standard & Poor's, and the average recovery rate – the proportion of bad debt that can be recovered – for those securities, which we assume is 40% based on historical observations of high-yield recovery rates.

Figure 21: Based on the building blocks above, the estimated return for US aggregate bonds is derived as follows:



Source: Invesco. US aggregate bonds are represented by the BBG BARC US Agg Bond Index. These estimates are forward-looking, are not guarantees, and they involve risks, uncertainties, and assumptions. These estimates reflect the views of Invesco Investment Solutions, the views of other investment teams at Invesco may differ from those presented here.

Figure 22: 10-year estimated fixed income market total returns (USD)

Asset class	Index	Estimated return %	Total yield %	Roll return %	Valuation change %	Credit loss %	Currency adjustment %
EM aggregate	BBG BARC EM Aggregate	3.80	5.69	0.31	-0.90	-1.30	0
Eurozone aggregate	BBG BARC Euro Aggregate	1.82	0.81	0.24	-1.25	-0.25	2.26
Global aggregate	BBG BARC Global Aggregate	2.05	1.96	0.25	-0.97	-0.18	0.98
US Treasury	BBG BARC US Treasury	1.70	2.30	0.25	-0.84	0	0
US HY corporates	BBG BARC US High Yield	4.50	6.96	0	-0.81	-1.64	0

Source: Invesco, estimates as of Sept. 30, 2019. All total returns data is annual. These estimates are based on our capital market assumptions which are forward-looking, are not guarantees, and they involve risks, uncertainties and assumptions.

Figure 23: US Aggregate Bond Index: CMA returns vs actual returns

Source: Invesco. Data from Jan. 31, 2000-Sept. 30, 2019. An investment cannot be made directly into an index. Capital market assumptions are forward-looking, are not guarantees and they involve risks, uncertainties and assumptions. These estimates reflect the views of Invesco Investment Solutions, the views of other investment teams at Invesco may differ from those presented here.

04

Alternatives: Private Assets

A significant, and increasing, number of institutional portfolios contain private or alternative assets[1]. This trend is likely due to shrinking expected returns and yields in traditional public assets. Private asset market capitalization has grown to $5.8T globally in 2019[2], Private Equity, a large subset, has experienced 7.5x growth since 2002 compared to 3x in public markets[2]. Composed of a broad array of heterogeneous investments, private assets are anything but standardized. As the space is evolving to include new assets and creates unique challenges to investors, we attempt to assess the economics of some common types of investments. In this portion of the CMA methodology we will present our views on; Private Equity, specifically Leveraged Buyouts (LBO), Private Direct Real Estate (DRE), and both Private Infrastructure Equity and Debt. To properly introduce private assets into a portfolio, we suggest taking a building blocks based approach to understand and forecast return, which we will address in detail in the following sections.

Notable differences between public and private assets are:

+ **Illiquidity.** Should one sell a private asset before its maturity, there are likely capital market frictions and significant penalties resulting in loss of principal. For this exercise, we assume all assets are held until their deal's expiration date and are calculated as a single period internal rate of return, differing from our approach to public markets, which represent the average of multiple periods of underlying investments.
+ **Leverage.** Private asset firms add leverage to portfolio assets to fund any required restructuring. This additional funding acts as a multiplier to any traditional capital gains or losses, accelerating the change in earnings and multiple expansion. Additionally, we estimate the unlevered versions of private assets.
+ **Fees.** Cost of financing from a leveraged buyout debt issuance and performance-based ("2 & 20") management fees are examples of large negative detractors to final return that do not typically exist in public assets. All of our private CMA's include an estimate net of fees, some explicitly and some implicitly. Since fees vary tremendously in private assets, we model it as an assumption that can be adjusted from client to client.

Model flexibility and the private benchmark problem

While an investor in public assets can simply buy an index of an asset class, own a portion of the universe, and experience average results, an investor in private assets cannot. To align our private CMAs with our public CMAs, but still provide the custom nature private asset classes require, we built enough flexibility into our private asset models to analyze the whole market, an individual fund, or a single deal.

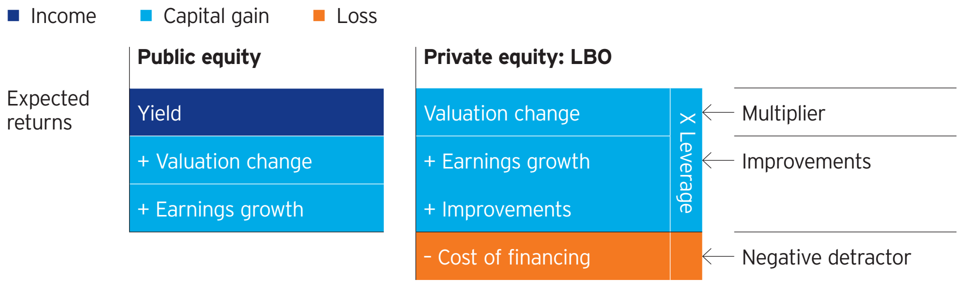
To emphasize the underlying reason for a customizable private model, there is simply no investable benchmark for private assets. These assets are unlisted on any tradeable market, provide at-best quarterly reporting or tender dates, and lack transparency of the underlying investments required to create a proper benchmark.

1 Lerner 2018
2 McKinsey; Preqin, 2019

Private equity: Leveraged buyouts

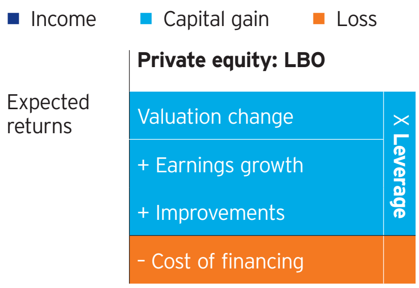
Our LBO estimates model the expected performance of a private equity (PE) firm in purchasing a public market company and taking it private through the realization of the investment over a 10-year investment horizon. Our estimates reflect an Internal Rate of Return (IRR) to a Limited Partner (LP) in the fund, while a General Partner (GP) would experience returns gross of fees. We derive our inputs from both the public markets for valuation multiples and fundamental corporate data as well as from peer-reviewed academic studies (Hooke et al., 2016, and Axelson et al., 2013) regarding deal structures, operational improvements, and firm leverage.

Figure 24: Comparing the building blocks of private assets to public assets



For illustrative purposes only.

Leverage



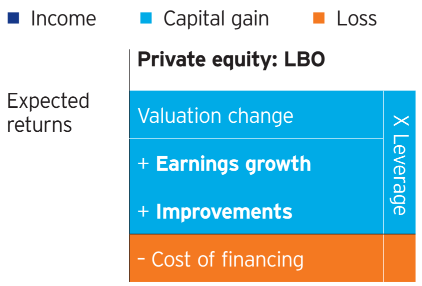
For illustrative purposes only. LBO = Leveraged buyouts

Embedded in the name of a leveraged buyout is the leverage component that PE firms use to finance deals. Once a debt-to-equity ratio, or leverage level, is targeted, firms maximize the debt used over the life of their deal, achieving that ratio. We assume PE firms have a target debt level of nearly 4x times the pre-takeout leverage for portfolio companies of, around 70-90% debt to value (Axelson et al. [2010], Jonathan Cohn [2013], Axelson et al. [2007a], Guo et al. [2008]). This additional leverage increases the value of the tax shield as well as the cost of financing.

We estimate the amount of additional leverage a firm can use in the take-out in order to achieve the targeted leverage ratio. This added leverage changes the value of debt as a percentage of the enterprise value as well as the interest expense as a component of the full firm's earnings.

References:
Jeffrey Hooke, Ken C. Yook, and Stephen Hee. The performance of mostly liquidated buyout funds, 2000-2007 vintage years. Available at SSRN, April 2016.
Steven Kaplan and Per Stromberg. Leveraged buyouts and private equity. Journal of Economic Perspectives, Volume 22, Number June 4, 2008.
Ulf Axelson, Tim Jenkinson, Per Stromberg, and Michael S Weisbach. Borrow cheap, buy high? the determinants of leverage and pricing in buyouts. Working Paper 15952, National Bureau of Economic Research, April 2010.
Erin Towery Jonathan Cohn, Lillian Mills. Evolution of capital structure and operating performance after leveraged buyouts: Evidence from us corporate tax returns. Available at authors website, McCombs School of Business, UT Austin, April 2013.
Shourun Guo, Edie S Hotchkiss, and Weihong Song. Do buyouts (still) create value? Working Paper 14187, National Bureau of Economic Research, July 2008.

Earnings growth and Improvements



For illustrative purposes only. LBO = Leveraged buyouts

Our private markets earnings estimate model differs meaningfully from our public markets approach. Implicit in our public market models is that the capital structure does not meaningfully change from the moment an asset is purchased through the investment horizon. However, LBOs immediately violate this assumption, so our LBO earnings model is also different.

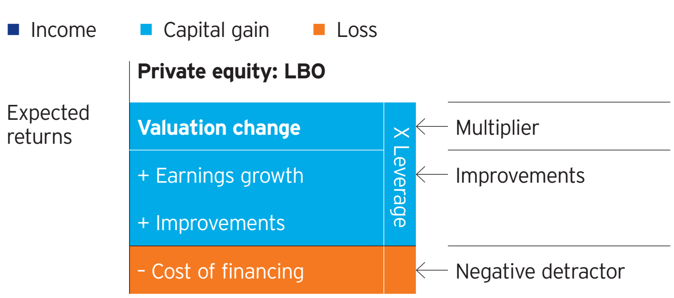
We estimate the full-firm earnings (EBITDA) fundamentally by decomposing EBITDA into its components:

EBITDA = Net Income + Tax Expense + Interest Expense + Depreciation/Amortization

Using a subsample of the public market universe, we estimate the current Net Income multiple and US corporate tax rates for the tax expense. By incorporating the PE firm's target capital structure into our estimates for the interest expense, we account for the leverage passed on to LBO targets; the estimate is a combination of the debt-to-equity ratio, expected interest rate from our CMAs and ROE estimates. The depreciation/amortization estimate is based on a straight-line depreciation and public-market tangible asset data.

We assume PE firms improve a company's operations above comparable public firm's, which leads to improved earnings growth. This measurement includes the effects of an increase in the value of the tax-shield resulting from added leverage. Behind the scenes, the purchasing firm's new management can influence a company's restructuring. Firms can write down the value of impaired assets and implement or other strategic initiatives to unlock untapped value. We include an increase of 10% (Kaplan and Stromberg [2008], Guo et al. [2008]) over public-market CMA earnings growth estimates, on top of any tax-shield related benefits due to added leverage.

Valuation change



For illustrative purposes only. LBO = Leveraged buyouts

References:
Steven Kaplan and Per Stromberg. Leveraged buyouts and private equity. Journal of Economic Perspectives, Volume 22, Number June 4, 2008.
Shourun Guo, Edie S Hotchkiss, and Weihong Song. Do buyouts (still) create value? Working Paper 14187, National Bureau of Economic Research, July 2008.

PE firm investments are subject to market valuation movements in the same manner as public investments as private valuations reflect their public market counterparts. While certain investments may be particularly attractive to PE firms (companies with high earnings yield and stable cash flows, for example), firms entering investments when valuations are high can suffer a drag on their performance as multiples return to more normal levels. We model this effect by estimating an expected multiple from trailing historical data.

$$\text{Valuation change} = \left({}^{EV}/EBITDA_F - {}^{EV}/EBITDA_C\right) \times \text{Expected EBITDA}$$

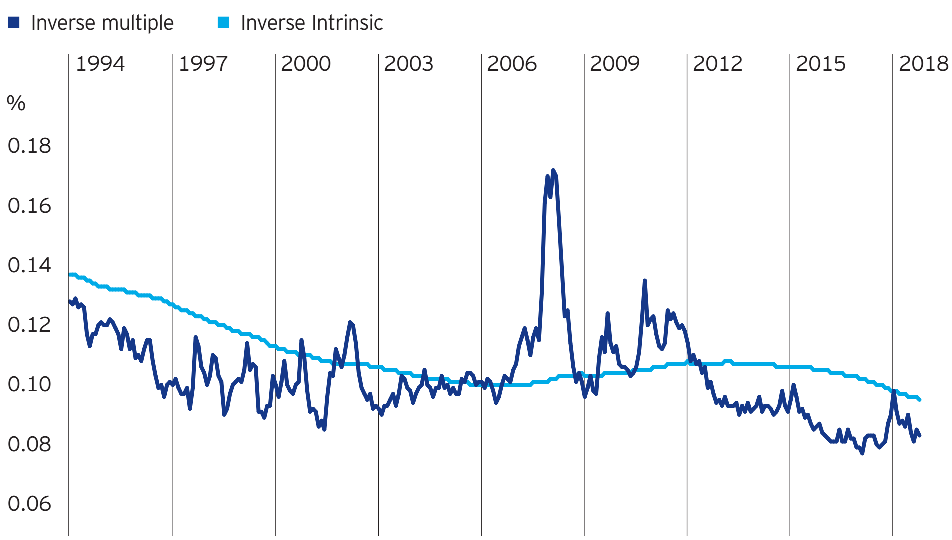
EBITDAc = Current public market valuation (subsample from the Russell 3000); EBITDAf = Trailing 10-year average of the Current Multiples; Expected EBITDA = We estimate this value from fundamental data.

Notably our multiples for private firms differ from our public CMAs in that we use full firm multiples (EV/EBITDA) of a publicly available universe of likely buyout targets (a subset of firm in the Russell 3000), rather than equity valuations (Price / Equity) to account for the total ownership of the portfolio company.

LBO valuations are mean-reverting, and when current multiples are high, like at the end of 2008, multiple contractions should be expected. High current multiples or overpayment lead to a reduction in future returns when the portfolio company is finally sold.

We also incorporate an adjustment to public market equity multiples to account for a takeout premium of the equity in order for the PE firm to acquire the target. Our estimate of 25% above public value is rooted in academic research of historical deal premia (Kaplan and Stromberg 2008). This adjustment biases our valuation changes downward to account for a premium at purchase but unnecessary when the portfolio company is either resold or relisted in public markets.

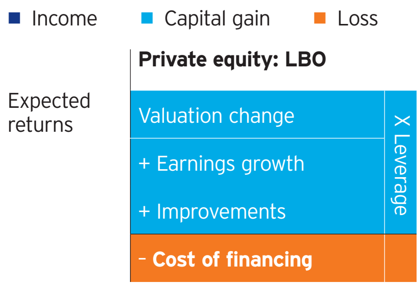
Figure 25: Mean-reverting nature of LBO multiples compared to their intrinsic value



Source: Invesco Investment Solutions Proprietary Research, FactSet Research Systems Inc. Sept. 30, 2019

References:
Steven Kaplan and Per Stromberg. Leveraged buyouts and private equity. Journal of Economic Perspectives, Volume 22, Number June 4, 2008.

Cost of financing



For illustrative purposes only. LBO = Leveraged buyouts

The cost of financing, or fair value of leverage, is modeled as the product of book yield, applied leverage, and return on debt. Other than the mechanics of a changing capital structure from added debt, the current cost to borrow versus the future cost to borrow can impact a firm's ability to add debt when needed, and thus the underlying deal. Higher current costs relative to future yields are a drag on expected returns. We use expected yields from our public CMA estimates of US high yield bonds to estimate current and prevailing borrowing rates.

Figure 26: 10-year estimated Private Equity LBO market total returns (USD)

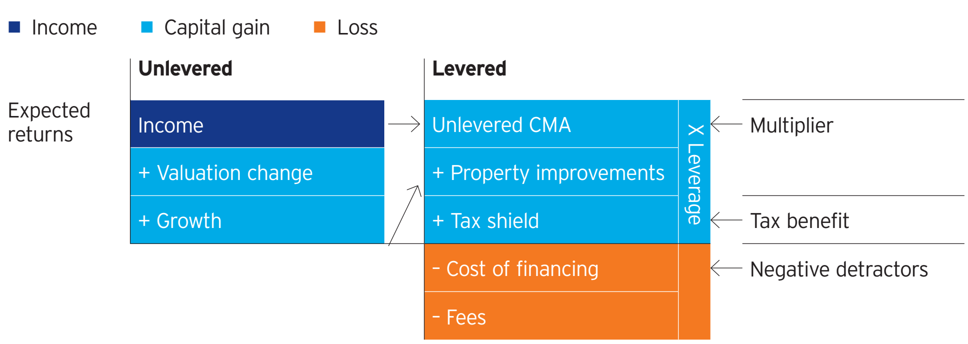
Asset class	Estimated return %	Earnings growth + improvement %	Valuation change %	Cost of financing %
Private Equity: LBO	12.29 =	22.83	- 0.77	-9.41

Source: Invesco, estimates as of Sept. 30, 2019. All total returns data is annual. These estimates are based on our capital market assumptions which are forward-looking, are not guarantees, and they involve risks, uncertainties and assumptions.

Private Direct Real Estate

The structure of Direct Real Estate (DRE) investments differ from their public counterpart in Listed REITs, in that REITs trade similarly to listed equity and have been shown empirically to show a positive correlation to listed equities over time with similar levels of volatility. On the contrary, private Real Estate exhibits a lower correlation to listed equity along with lower volatility. Listed REITs will often use leverage to amplify returns, which also amplifies volatility. We model private real estate on an unlevered basis first and then allow leverage to enter the equation after we have determined the return associated with the unlevered asset. A building block framework for CMAs that focuses on income and capital appreciation makes the unlevered DRE asset class model comparable to that of other asset classes, then easily scales to the levered version once one accounts for leverage, cost of financing, and tax benefits.

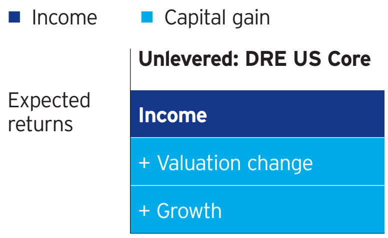
Figure 27: Comparing the building blocks of Unlevered Private Direct Real Estate (DRE) to Levered DRE



For illustrative purposes only.

Unlevered: Direct Real Estate US Core

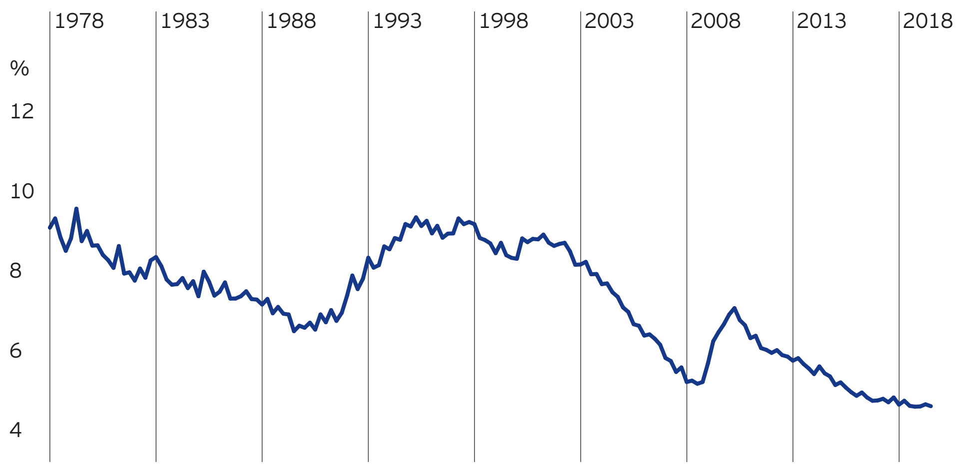
Income



For illustrative purposes only.

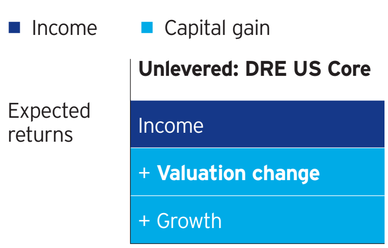
Starting with the capitalization rate (cap rate), a proxy for rental income from the NCREIF Property Index (NPI), we subtract expected capital expenditures required to maintain a property, of 1.5%, which is slightly less than the 2% reported in academic research (Gosh and Petrova, 2017). Cap rates for US Core Real Estate have been similarly falling since the 1980's in a similar fashion to most yields globally, from 9.5% to 4.5% today.

Figure 28: NCREIF Property Index Cap Rate from 1978 to 2019



Source: Invesco, estimates as of Sept. 30, 2019.

Valuation Change



For illustrative purposes only.

References:
Ghosh and Petrova 2017, The impact of capital expenditures on property performance in commercial real estate, The Journal of Real Estate Finance and Economics 55, 106-133

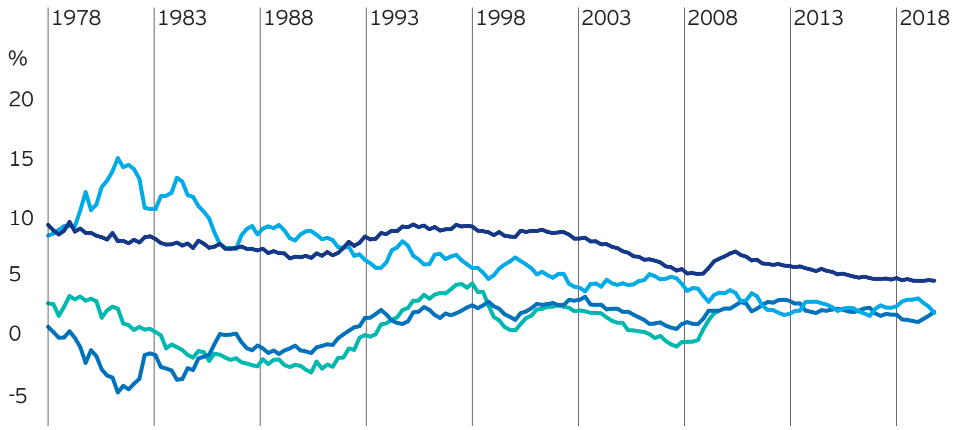
To isolate US Core Real Estate's valuation change, we start with the NCREIF capital return index and remove Capex, real NOI growth and inflation. We found a relationship between valuations, cap rates, and US rates as follows, especially using the data after 1990.

Valuation model: $\hat{V}_{t \rightarrow t+t+10} = 0.70 \text{ x } (RF_{Cap,t} - \widetilde{RF}_t)$

RF_{Cap} = Cap rate; $\widetilde{RF}$ = 10-year Treasury nominal rate

Figure 29: Predicted US core direct real estate valuation change model and inputs

■ Cap rate ■ 10-year Treasury rate ■ CMA valuation model ■ Realized 10-year valuation change



Source: Invesco, estimates as of Sept. 30, 2019. These estimates are forward-looking, are not guarantees, and they involve risks, uncertainties, and assumptions. These estimates reflect the views of Invesco Investment Solutions, the views of other investment teams at Invesco may differ from those presented here.

Growth

■ Income ■ Capital gain

	Unlevered: DRE US Core
Expected returns	Income
	+ Valuation change
	+ Growth

For illustrative purposes only.

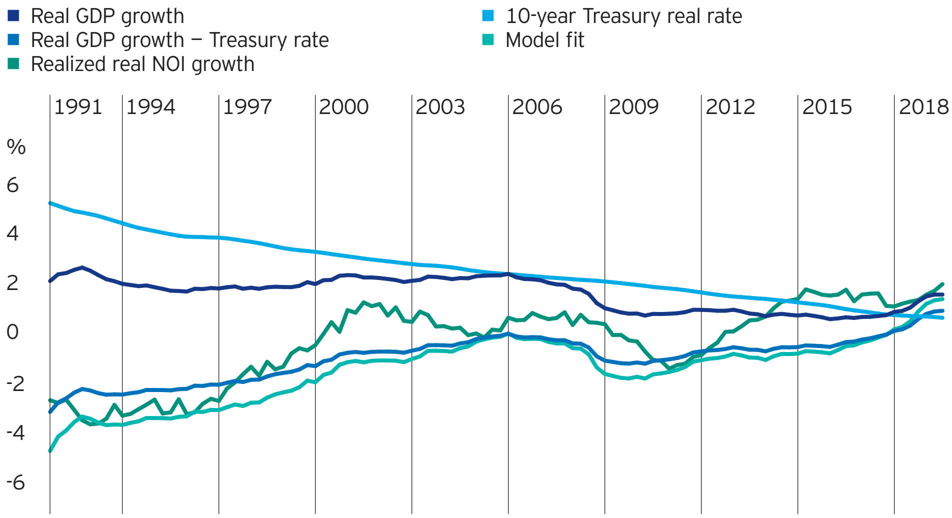
To identify expected real rental income growth, or net operating income, we multiply the difference of expected real GDP growth with that of US interest rates by 1.5, the Beta we identified of the model's inputs to future income growth. The coefficient is estimated by studying the relationship between realized NOI growth in NPI index with realized GDP growth and treasury rate. We also use the NOI growth number in the NCREIF-ODCE index as a robustness check.

Growth model: $\hat{G}_{NOI,t} = 1.5 \text{ x } (G_{GDP,t} - RF_t)$

$G_{GDP,t}$ = Real GDP growth rate; RF_t = 10-year Treasury real rate

Finally, to get a nominal growth rate, we add inflation expectations estimated by the Cleveland Fed.

Figure 30: Real NOI growth model and inputs



Source: Invesco, estimates as of Sept. 30, 2019.

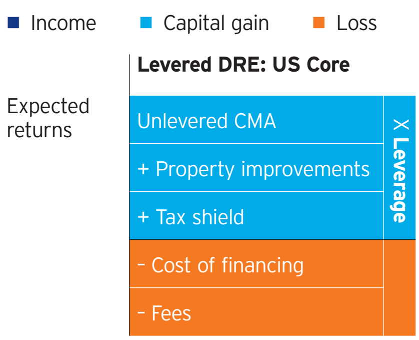
Figure 31: 10-year estimated US core direct real estate unlevered market total returns (USD)

Asset class	Estimated return %	Income %	Valuation change %	Growth %
US Core Direct Real Estate Unlevered	7.60	3.02	1.97	2.61

Source: Invesco, estimates as of Sept. 30, 2019. All total returns data is annual. These estimates are based on our capital market assumptions which are forward-looking, are not guarantees, and they involve risks, uncertainties and assumptions.

Levered: Direct Real Estate US Core

Leverage



For illustrative purposes only.

Starting with the unlevered return and adding in a property improvement assumption of 2% as this term captures the value add or alpha a manager provides in DRE (Lee, Shilling, and Wurtzebach 2016), we can begin to estimate a levered version of the DRE US Core model. Once a loan is financed, we use the loan-to-value (LTV) ratio to estimate the amount of leverage being applied and use it to scale the unlevered return CMA. As taxes are paid only on the real estate's value but not on the loan, we add back in a tax benefit based on current tax rates and amount of leverage applied to the loan. The current corporate tax rate of 21% is applied to derive the size of the benefit. Finally, we subtract a cost of capital which, we estimate from our commercial mortgage-backed securities (CMBS) CMA with a duration of around five years. As financing costs increase, the difference between the levered and unlevered return shrinks.

References:
Lee, Jin Man, James D. Shilling, and Charles Wurtzebach. "A New Method to Estimate Risk and Return of Commercial Real Estate Assets from Cash Flows: The Case of Open-End (Diversified) Core Private Equity Real Estate Funds." (2016).

Given a certain leverage level, the levered CMA return is calculated as follows:

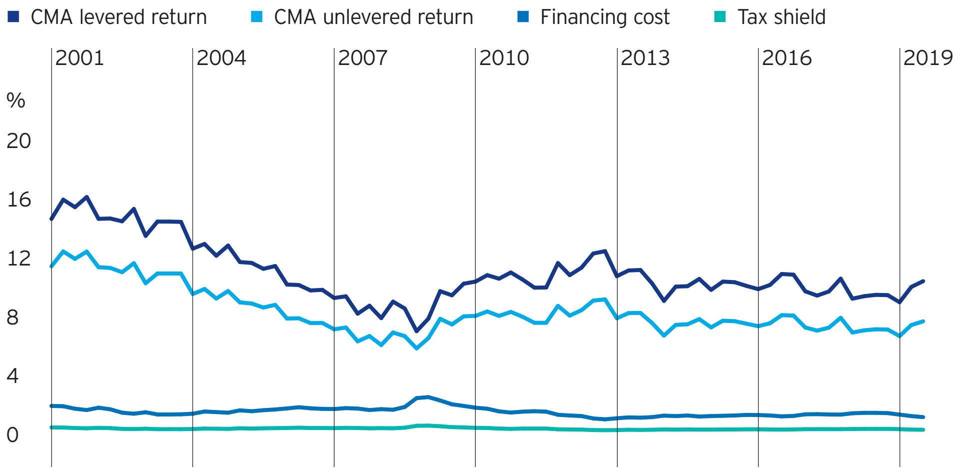
$$R_{DRE,\ Levered} = (R_{DRE,\ Unlevered} + Property\ Improvement) \times \frac{1}{1 - LTV} - R_{Cost\ of\ Capital} \times \frac{LTV}{1 - LTV} + Tax\ shield - Fees$$

$R_{DRE,Levered}$ = Levered CMA return; $R_{DRE,\ Unlevered}$ = Unlevered CMA return; Property Improvement = Assumed to be 2%; LTV = Loan to value ratio assumed to be 22.5%; $R_{Cost\ of\ Capital}$ = Expected (CMBS) rate from public CMA;

Tax Shield = $R_{Cost\ of\ Capital} \times \frac{LTV}{1 - LTV} \times$ Corporate Tax Rate (21%);

Fees = Assumed management fee of 1.2% (Source: Invesco GDRE).

Figure 32: US Core DRE CMA Return with leverage and without



Source: Invesco, estimates as of Sept. 30, 2019. These estimates are forward-looking, are not guarantees, and they involve risks, uncertainties, and assumptions. These estimates reflect the views of Invesco Investment Solutions, the views of other investment teams at Invesco may differ from those presented here.

Figure 33: 10-year estimated US core direct real estate levered market total returns (USD)

Asset class	Estimated return %	Unlevered return %	Property improvement %	Tax shield %	Cost of financing %[1]	Fees %
US Core Direct Real Estate Levered	10.33 =	7.60	+ 2.00	+ 0.25	-1.09	-1.20

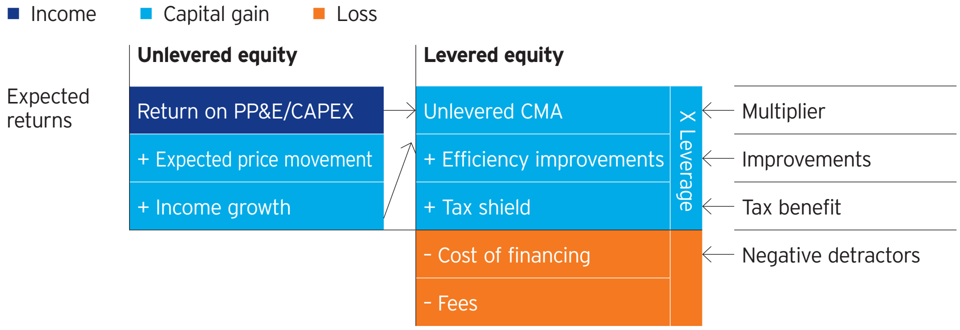
Source: Invesco, estimates as of Sept. 30, 2019. All total returns data is annual. These estimates are based on our capital market assumptions which are forward-looking, are not guarantees, and they involve risks, uncertainties and assumptions.

1 Cost of Financing = $R_{Cost\ of\ Capital}$ * LTV / (1-LTV)

Private Infrastructure: Equity

Over the past two decades, the degree in which private capital is able to participate in the financing and operation of public infrastructure developments has grown substantially (Preqin, 2019). Due to expectations around growing populations, the enhanced infrastructure of previously less developed economies, and the replacement of aging assets globally this asset class is expected to continue its growth pattern over the next decade (IMF, 2019). As a result, we expect an expanding number of investors to show an interest in the infrastructure space.

Figure 34: Comparing the building blocks of Unlevered Private Infrastructure Equity to Levered Private Infrastructure



For illustrative purposes only.

The building blocks for Private Infrastructure equity - levered are:

+ **Return on PP&E.** A property's income is its return on property plant and equipment (PP&E), which is calculated as Net Operating Income (NOI) divided by Net PP&E, or how much income is generated for every dollar invested in the asset. Our universe is the Dow Jones Brookfield Global Infrastructure Index. We subtract out estimated Capital Expenditures (Capex), found by identifying the median net useful life of properties outstanding in our universe, of 26 years and from there estimating the median maintenance costs.

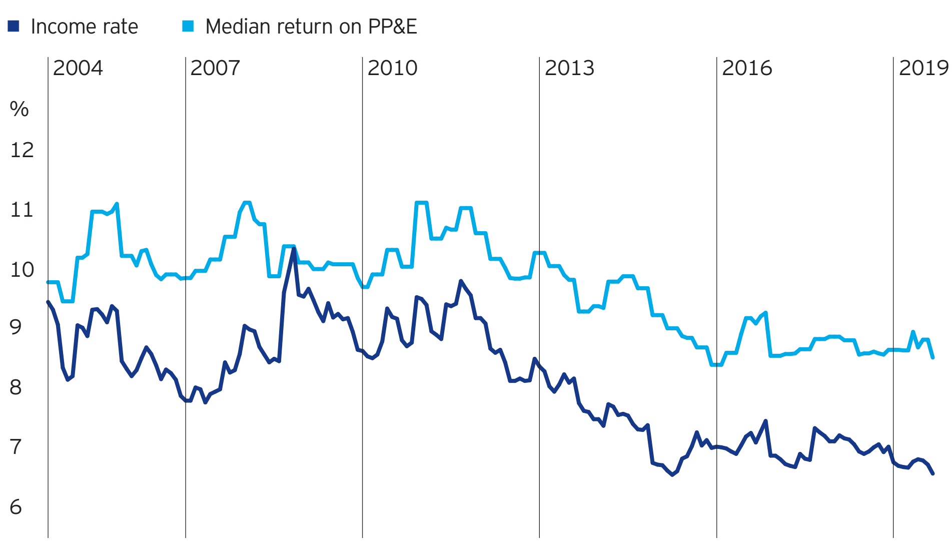
The formula for Return on PP&E for the median infrastructure property is stated as follows:

Return on PP&E (book value of property) = Operating Income/Net PP&E

Operating Income = Gross Income -Operating Expenses; Net PP&E = Gross PP&E + Capex -Accumulated Depreciation

A market value adjustment (Enterprise Value/Assets Ratio) is applied to discount PP&E to a new value, the Income Rate, which is the market value of the property (**Figure 35**).

Figure 35: Comparing median infrastructure PP&E to income rate



Source: Invesco, estimates as of Sept. 30, 2019.

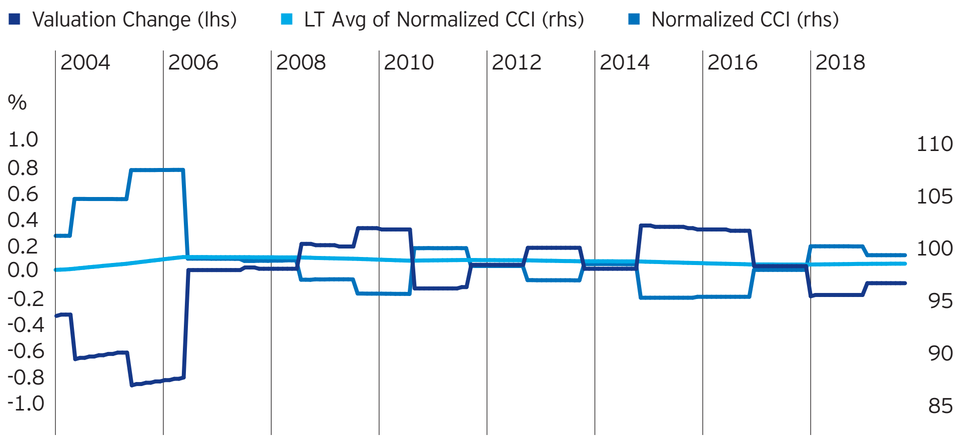
References:
International Monetary Fund. 2019. World Economic Outlook: Global Manufacturing Downturn, Rising Trade Barriers. Washington, DC, October.

+ **Valuation change.** We expect asset prices to rise with the cost of construction, which we model by normalizing the Construction Analytics' Construction Cost Index (CCI) by GDP. Overvaluation, represented by positive deviations from the long-term average, represents potential decreases in future returns.

One can calculate Valuation change using the following formula:

Valuation change = (long-term average of normalized CCI / current normalized CCI)$^{(1/10)}$ – 1

Figure 36: Expected price movement for Private Infrastructure modeled by normalized CCI



Invesco, estimates as of Sept. 30, 2019.

+ **Income Growth.** US infrastructure properties NOI growth is estimated by nominal GDP.

Figure 37: 10-year estimated private infrastructure equity – unlevered market total returns (USD)

Asset class	Estimated return %	Income %	Valuation change %	Growth %
Private Infrastructure Equity - Unlevered	6.99 =	2.70	-0.08	4.37

Source: Invesco, estimates as of Sept. 30, 2019. All total returns data is annual. These estimates are based on our capital market assumptions which are forward-looking, are not guarantees, and they involve risks, uncertainties and assumptions.

The building blocks for Private Infrastructure equity, levered, are similar to that of Private Direct Real Estate.

Given a certain leverage level, the levered CMA return is calculated as follows:

$$R_{DRE,\ Levered} = (R_{DRE,\ Unlevered} + Efficiency\ Improvements) \times \frac{1}{1 - LTV} - R_{Cost\ of\ Financing} \times \frac{LTV}{1 - LTV} + Tax\ shield - Fees$$

Where:

+ **Efficiency improvements.** Private asset managers are assumed to improve Return on PP&E from median to the third quintile level in the infrastructure universe.
+ **Leverage.** Once a loan is financed, we use the loan-to-value (LTV) ratio of 33%, the median leverage of the universe, to estimate the amount of leverage being applied and use it to scale the unlevered return CMA.
+ **Tax Shield.** The tax rebate on assets purchased by debt applies to all levered assets.
+ **Cost of financing.** Like that of private equity or direct real estate, the cost of financing is a negative component of expected returns for levered private infrastructure. The CMA yield on private investment-grade global infrastructure debt (**Figure 38** in the following section) is our choice to estimate the current cost to fund these assets.
+ **Fees.** Management fees are calculated as a flat fee of 150 bps, the median of the funds within the private global infrastructure category from Preqin, and 20% carried interest.

Figure 38: 10-year estimated private infrastructure equity — levered market total returns (USD)

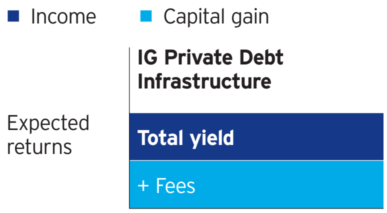
Asset class	Estimated return %	Unlevered return %	Property improvement %	Tax shield %	Cost of financing %	Fees %
Private Infrastructure Equity - Levered	8.09 =	6.99	+ 1.45	+ 0.39	-1.87	-3.09

Source: Invesco, estimates as of Sept. 30, 2019. All total returns data is annual. These estimates are based on our capital market assumptions which are forward-looking, are not guarantees, and they involve risks, uncertainties and assumptions.

Private Infrastructure: Debt

The building blocks for Private Infrastructure Debt - Investment Grade are:

Total yield



For illustrative purposes only.

Like a public bond, a yield estimate (**Figure 39**) is the key driver of return for private infrastructure debt. The major difference is that the current yield is the spread of global infrastructure yield over global treasuries plus LIBOR, as most of the debt is floating rate. Structurally, unlisted debt is not traded and thus not exposed to yield curve movements like rolldown or valuation changes.

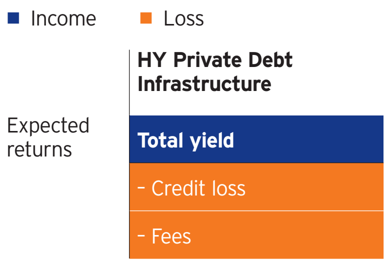
Figure 39: 10-year estimated Private Infrastructure Debt IG market total returns (USD)

Asset class	Estimated return %	Total yield %	Roll return %	Valuation change %	Credit loss %	Currency translation %
Private Infrastructure Debt IG	3.75 =	3.10	+ 0.00	+ 0.00	-0.00	+ 0.68

Source: Invesco, estimates as of Sept. 30, 2019. All total returns data is annual. These estimates are based on our capital market assumptions which are forward-looking, are not guarantees, and they involve risks, uncertainties and assumptions.

The building blocks for Private Infrastructure Debt – High Yield are:

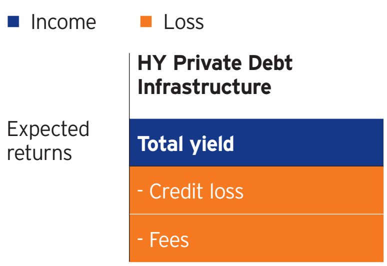
Total yield



For illustrative purposes only.

Taking a similar approach as investment-grade private infrastructure debt in estimating total yield, the only difference is the current yield where global infrastructure high-yield spreads are taken over global AAA yields.

Credit loss



For illustrative purposes only.

Minimal losses are anticipated even in high-yield infrastructure as an estimated 2.5% of all issues default with a 73% recovery rate. This is a higher rate than traditional high yield due to the asset backed nature of the debt.

Figure 40: 10-year estimated Private Infrastructure Debt HY market total returns (USD)

Asset class	Estimated return %	Total yield %	Roll return %	Valuation change %	Credit loss %	Currency translation %
Private Infrastructure Debt HY	5.72 =	5.54	+ 0.00	+ 0.00	-0.68	+ 0.43

Source: Invesco, estimates as of Sept. 30, 2019. All total returns data is annual. These estimates are based on our capital market assumptions which are forward-looking, are not guarantees, and they involve risks, uncertainties and assumptions.

05

Alternatives: Hedge Funds and Listed Real Assets

Estimating returns for such investments is more complex than evaluating equities and fixed income, as the range of alternatives ("alts") available runs the entire spectrum of risk. Long/short strategies, for example, behave differently than commodities, and both behave differently than global macro. And for any alternative category, it can be a challenge to know how much of the return is true, uncorrelated alpha, and how much can be attributed to broad market exposures (e.g., S&P 500 Index). In fact, academic research (Hasanhodzic and Lo, 2007; and Fung and Hsieh, 2004) suggests that a significant portion of hedge fund returns is attributable to conventional asset class and factor risks. Leaning into this research, we construct linear models using available market indexes from our traditional asset class CMAs and measure the proportion of the estimated returns and volatility that are attributable to them.

Our capital market assumptions consider hedge fund asset classes and listed real estate investment trust asset classes. For each of these, we perform a regression-based analysis that seeks to decompose returns as follows:

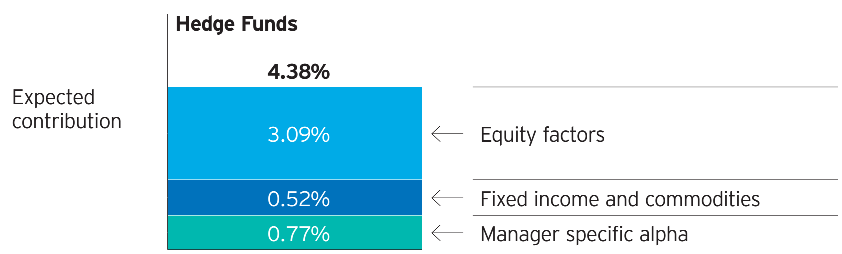
Figure 41: Decomposed hedge fund returns through a factor model

$$Hf_i = a + \sum_j bX_i$$

i = hedge fund index; j = market/conventional asset class risk factor; *j* = US Large Cap, US Mid Cap, US Small Cap, International Developed Equities, Emerging Market Equities, US Treasuries, US Investment Grade Bonds, US High Yield Bonds, International Fixed Income, Emerging Market Bonds, and Commodities.

All returns are orthogonalized based on Chow and Klein (2013), which examines the impact of individual market exposures on the return variation of risky assets. Coefficients are estimated using rolling 84-month Stepwise regressions. The regression results decompose hedge fund index returns into systemic risk (beta) and idiosyncratic risk (manager-specific alpha).

Figure 42: Estimating contribution to hedge fund returns



Source: Invesco Investment Solutions Research, Sept. 30, 2019.

Figure 43: 10-year estimated hedge fund total returns (USD)

Asset	Index	Expected return %	Systematic return %	Alpha %
CISDM CTA	CISDM CTA Index	4.65	0.29	4.36
CISDM Global Macro	CISDM Global Macro Index	1.93	0.39	1.54
CS Managed Futures	Credit Suisse Managed Future Index	2.04	0.19	1.85
Hedge Funds	HFRI HF Index	4.38	3.61	0.77
HF Event Driven	HFRI Event Driven Index	5.45	3.67	1.78
HF Market Neutral	HFRI Equity Market Neutral Index	2.33	0.85	1.48

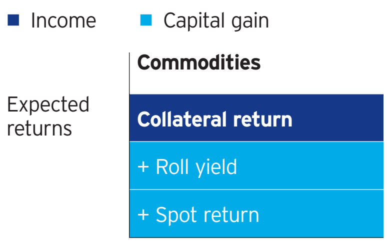
Source: Invesco, estimates as of Sept. 30, 2019. All total returns data is annual. These estimates are based on our capital market assumptions which are forward-looking, are not guarantees, and they involve risks, uncertainties and assumptions.

Commodities

To estimate commodities returns we analyze the futures curve, which is a graphical representation of commodity contracts (agreements to buy or sell a predetermined amount of a commodity at a specific price on a specific date in the future) that expire at different maturities. As with other asset classes, we apply the building block approach to the futures curve to identify yield (collateral return) and appreciation (roll return and spot return) as the main constituents of total return.

Within the asset class, we apply this methodology consistently across the individual commodity sectors that make up the main commodity indices, the S&P GSCI Index and the Bloomberg Commodity Index including Agriculture, Energy, Industrial metals, Livestock, and Precious metals.

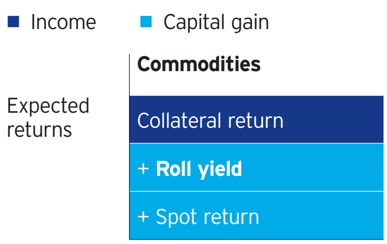
Collateral return



For illustrative purposes only.

Collateral return is meant to reflect the value of the return on cash, which is needed as collateral for trading in commodity futures. The return is a function of the fixed income instrument in which the cash is invested – for example, short-term US T-bills. We use an average of the current US three-month T-bill interest rate and 10-year forecasted US three- month T-bill interest rate from the Federal Reserve Bank of Philadelphia to estimate this value.

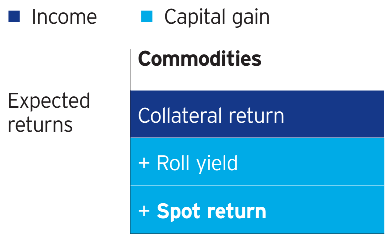
Roll yield



For illustrative purposes only.

Roll yield reflects the return from rolling the commodity futures forward – in other words, from wanting to maintain exposure to a commodity after the contract has expired. It reflects the potential return from the movement in the price of the futures contract toward the spot price over time. We estimate roll yield through the difference between historical excess returns, which includes roll return and the historical spot return, which measures only the price return.

Spot return



For illustrative purposes only.

The spot return attempts to capture the return that can be derived from an increase in the value of a commodity as a real asset, beyond its ability to capture the value of keeping up with long-term inflation. For the purposes of estimating the real spot return, we use the long-term historical average of real spot monthly returns dating back to 1970 and adjust that for expected inflation over a 10-year time horizon.

Figure 44: 10-year estimated commodities total returns (USD)

Asset class	Index	Expected return %	Real spot return %	Expected inflation %	Roll return %	Collateral return %
Agriculture	S&P GSCI Agriculture	0.74	-0.02	1.56	-2.96	2.16
Energy	S&P GSCI Energy	7.80	1.43	1.56	2.63	2.16
Industrial Metals	S&P GSCI Industrial Metals	5.12	1.73	1.56	-0.35	2.16
Livestock	S&P GSCI Livestock	2.59	-0.33	1.56	-0.79	2.16
Precious Metals	S&P GSCI Precious Metals	3.22	3.59	1.56	-4.14	2.16
Commodities	S&P GSCI	5.43	1.12	1.56	0.58	2.16
BB Commodities	Bloomberg Commodity	4.15	1.26	1.56	-0.85	2.16

Source: Invesco, estimates as of Sept. 30, 2019. All total returns data is annual. These estimates are based on our capital market assumptions which are forward-looking, are not guarantees, and they involve risks, uncertainties and assumptions.

06

5-year vs 10-year CMAs

In order to facilitate our efforts to engage in more "active-strategic" portfolio management, which involves the potential to actively position our strategic portfolios within the business cycle, we expanded our CMA methodology to support a shorter time horizon of five years. While still drawing on the building block approach that underpins the 10-year time horizon, the methodology for the five-year time horizon incorporates estimating elements that are appropriate for understanding the behavior of asset classes over a shorter holding period.

Equities: 5-year versus 10-year expected returns

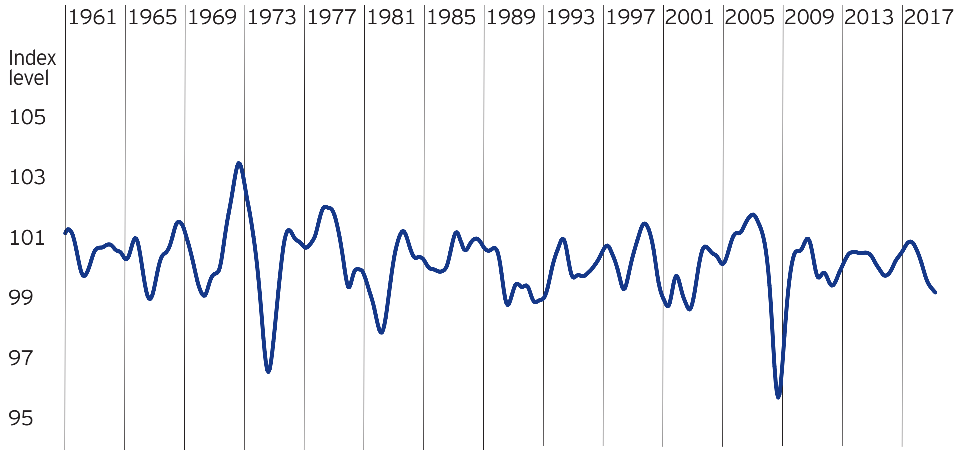
The building blocks for estimating equity returns for a five-year time horizon are generally the same as those identified for the 10-year time horizon — yield, earnings growth and valuation. However, the way in which each of these building blocks is constructed may change to better reflect shorter-term market dynamics.

+ **Yield.** Yield is estimated for the 10-year time horizon using the 10-year average total yield ratio, which reflects the impact of both dividend yield and buybacks. The same measure is used to estimate yield for the five-year time horizon.
+ **Earnings growth.** Long-term real GDP per capita provides a stable signal over time to estimate earnings growth across a 10-year time horizon. For a shorter time horizon, it needs to be adjusted to reflect a short-holding period.

Earnings growth = Long-term real GDP growth + Real GDP growth adjustment + Five-year expected inflation

According to academic research (Pritchett and Summers, 2014), economic growth rates globally have mean-reverting properties — meaning that future growth rates move in the opposite direction to current growth rates. This is particularly important in a five-year time horizon since we are not looking across the full economic cycle. We use the OECD Composite Leading Indicator (CLI) to gauge these short-term trends in economic growth. The CLI is designed to provide early signals of turning points in business cycles showing fluctuations of economic activity around its long-term potential level (which is normalized at 100).

Figure 45: Composite leading indicator (CLI) – OECD



Source: OECD. Data through Sept. 30, 2019.

Five-year real GDP growth = Long-term real GDP growth – b x (CLI - 100)

b = Relationship between short-term economic movements and forward five-year real GDP growth

For the regression run as of Sept. 30, 2019, US data indicated that short-term economic movements have led to an adjustment down of 0.30% in the forward five-year real GDP growth. Globally, we would expect that the pace of mean-reversion for each country would depend on its level of economic development. In other words, for countries that are considered "mature" or "developed" economies, and for which the rate of long-term growth is stable, we would expect a quicker reversion to the mean – and vice versa for emerging economies. For example, we expect that for Japan, which is considered a mature, slow-growing economy, short-term economic movements would revert more quickly to the long-term average. At the same time, we expect that in emerging markets, whose long-term growth rates are still evolving, short-term economic movements would revert less quickly to their amorphous long-term averages (see **Figure 46**). Although we expect these relationships to remain stable over the medium-term, we re-run the regressions and review the resulting data quarterly.

Figure 46: We expect the pace of mean-reversion to depend on a country's level of economic development

Region/Country	Pace of mean-reversion
United States	0.30
United Kingdom	0.28
Japan	0.50
Eurozone	0.39
Canada	0.33
Emerging markets	0.26
Asia Pacific ex-Japan	0.26

Source: OECD, Invesco as of Sept. 30, 2019.

+ **Valuation change.** Across a full business cycle, valuation change involved in estimating the potential for the current P/E level to revert to an estimated long-term average over a 10-year time horizon. Over a shorter time frame, we look at the potential for the P/E to revert back to the long-term average in five years' time.

Figure 47: Five-year vs 10-year capital market assumptions for US large-cap equities

Time horizon	Estimated return %	Yield %	Earnings growth %	Growth adjustment %	Valuation change %
10 years	5.63	3.00	4.19	–	-1.57
5 years	4.20	3.00	4.23	0.07	-3.10

Source: Invesco, estimates as of Sept. 30, 2019. All total returns data is annual. These estimates are based on our capital market assumptions which are forward-looking, are not guarantees, and they involve risks, uncertainties and assumptions.

Fixed income: 5-year versus 10-year expected returns

As with equities, the building blocks for estimating fixed income returns over a five-year time horizon are the same as those identified for the 10-year time horizon — yield, roll return, valuation change and credit loss. However, how each of these building blocks is defined may need to change to better reflect shorter-term market dynamics.

+ **Yield.** Return from yield reflects an average of the starting (current) and an estimate of the ending yield. For a five-year time horizon, we use an estimate of the five-year yield curve to evaluate ending yield, instead of the 10-year yield curve that we used for the long-term time horizon. To estimate the future yield curve, we use the same process, evaluating two specific points on the futures curve to help determine its level and shape. For the estimated five-year yield curve, we use the yield for the three-month Treasury bills and the yield for five-year Treasury notes. As previously discussed, another factor impacting the direction of potential future yield involves movement in credit spreads, which we estimate by looking at the relationship between current credit spreads and their 10-year rolling average.
+ **Roll return.** As previously discussed, the estimate for roll return reflects an average of the roll return from the current yield curve and the roll return from the ending (estimated) yield curve. As with the return from yield, instead of the 10-year estimated yield curve, we use the five-year estimated yield curve to calculate the average roll return.
+ **Valuation change.** The same methodology is used to estimate valuation change over a five-year time horizon as was used over a 10-year time horizon. The main difference, however, is that the impact on price from the shift to the ending yield curve is amortized over five years.
+ **Credit loss.** No change from the estimate used for the 10-year time horizon.

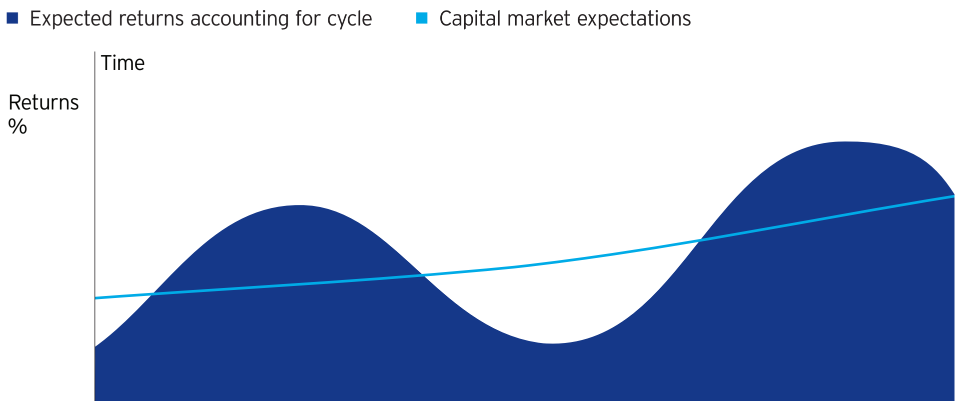
07

Tactical Asset Allocation

Strategic investors generally set asset allocation policy based on long-term expectations of asset class returns. However, asset prices do not evolve in a linear fashion. Instead, the performance of financial markets over the short-term is often driven by factors that may not be incorporated in the building blocks of a long-term CMA. For example, while long-term forecasts for growth and inflation inform CMAs, these variables exhibit pronounced cyclicality and fluctuations over the course of the business cycle, affecting asset prices in the short and medium-term. Similarly, while valuations provide long-term predictive power for asset returns, therefore informing long-term CMAs, evidence shows their performance as an indicator declines as the investment horizon shortens. As an example, using a common equities valuation measure such as earnings yield, or earnings divided by price, its predictive power over forward returns of S&P 500 shrinks from an R2, as a measure of best fit, of 50% over 10 years, to only 11% over 3 years, and less than 1% over 1 month. In short, different information influences different investment horizons.

Investors with relevant information about short-term price deviations may have an opportunity to benefit from price dislocations, but they must be willing to tactically shift away from policy. In this section, we present Invesco's tactical asset allocation methodology as a complementary framework to our long-term CMAs. We detail a macro-regime framework, which combines information from leading economic indicators and global market sentiment, to inform tactical asset allocation decisions over shorter time horizons, potentially allowing investors to seek additional return opportunities or navigate near-term risks.

Figure 48: Cyclicality of expected returns



For illustrative purposes only.

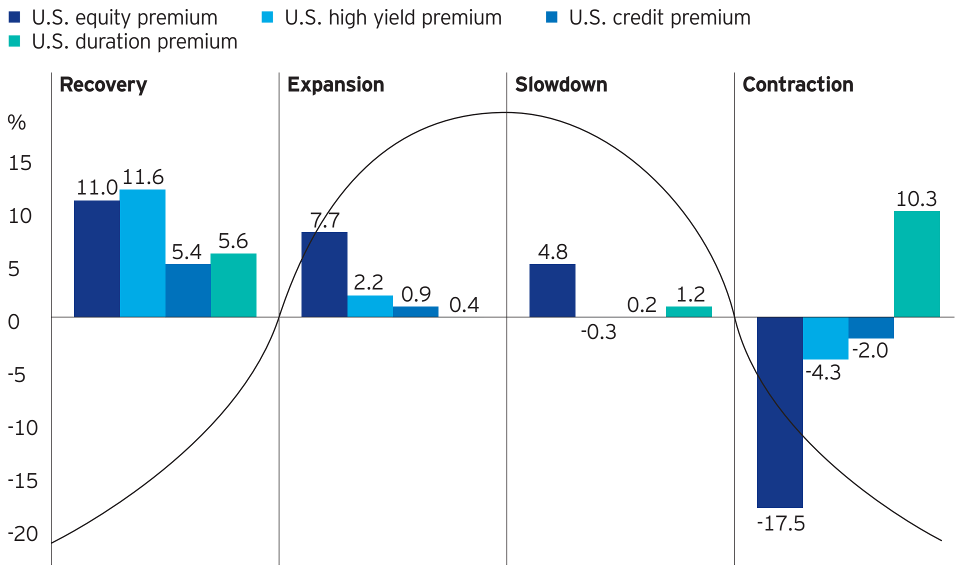
Invesco Investment Solutions (IIS) leading economic indicators

In our whitepaper, "Dynamic Asset Allocation through the Business Cycle" (*de Longis, 2019*), we develop a macro regime framework to forecast the performance of asset classes in different stages of the business cycle and provide empirical evidence of how prices of global equities, credit and sovereign fixed income are driven primarily by the change, not the level of economic growth. Using our macro framework, historical analysis of the last 50 years shows asset returns vary significantly between regimes, with major implications for asset allocation decisions. Furthermore, our results are consistent across regions, with the relative performance between asset classes exhibiting very similar patterns across markets.

We define the four stages of the business cycle based on the expected level and change in economic growth, measured via proprietary composite leading economic indicators:
+ Recovery, when growth is below trend and accelerating
+ Expansion, when growth is above trend and accelerating
+ Slowdown, when growth is above trend and decelerating
+ Contraction, when growth is below trend and decelerating

As illustrated in **Figure 49**, traditional risk premia (e.g., the relative performance between major asset classes) exhibit very different performance across the four stages of the cycle. On average, investors are compensated for taking extra risk (e.g., moving from safer to riskier asset classes) during a recovery or expansion phase of the cycle, when growth is accelerating. Conversely, in a slowdown regime, when growth is still above trend but begins to decelerate, the performance across asset classes starts to converge, with the equity risk premium still showing some compensation, albeit at lower magnitudes. In a contraction phase, investors are, on average, not rewarded for taking additional risk, and perceived safer assets such as government bonds typically offer superior returns.

Figure 49: Different risk premia have outperformed in different macro regimes
US historical returns (Jan. 31, 1970–June 30, 2019)



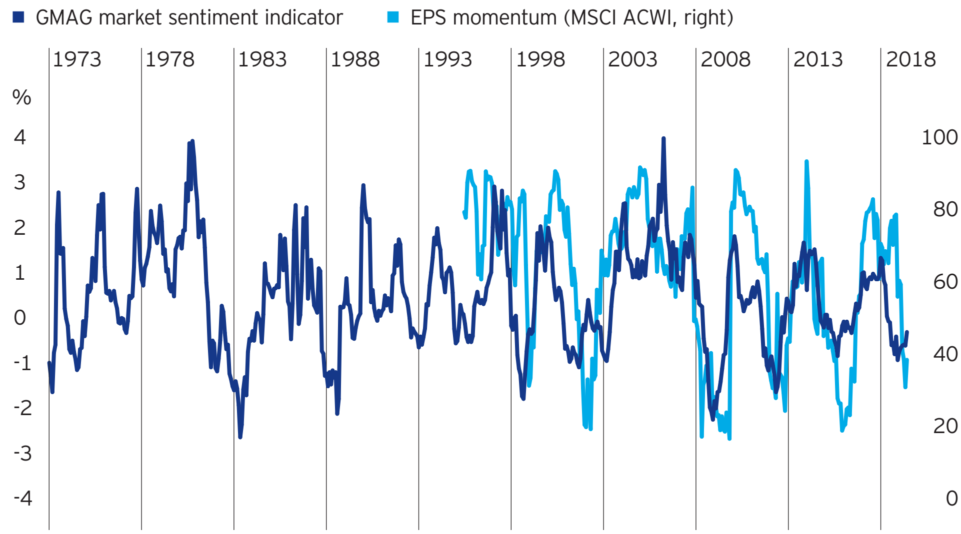
Source: Invesco. Proprietary research of the US Business Cycle Leading Indicator, June 30, 2019. Annualized monthly returns of the defined risk premia from Jan. 31, 1970–June 30, 2019. Risk Premia are defined as follows: US Equity Premium = S&P 500 - Citigroup US 7-10 YR Treasury. High Yield Premium = Citigroup High Yield Cash Pay BB Rated (7-10) YR - Citigroup USBIG Corp BBB Rated (7-10)YR. Credit Premium = Citigroup USBIG Corp BBB Rated - Citigroup US 7-10 YR Treasury. Duration Premium = Citigroup US 7-10 YR Treasury - Citigroup 90day T-Bill. **Past performance does not guarantee future results.**

IIS market sentiment indicator

Next to the information contained in leading economic indicators, we also analyze cyclical fluctuations in global asset prices to further support the identification of cyclical turning points in economic activity. In "Market Sentiment and the Business Cycle" (*de Longis and Ellis, 2019*), we outline a framework to extract market participants' expectations about future economic regimes and illustrate how global risk appetite can be used as a leading indicator and a real-time proxy of the global business cycle.

Our global market sentiment indicator provides a measure of relative risk-adjusted performance between riskier and perceived safer asset classes (e.g., equities vs. government bonds). Specifically, it measures how much investors have been rewarded on average, for taking an incremental unit of risk in global financial markets on a trailing medium-term basis. A rising index value signals improving market sentiment (i.e., rising risk appetite). Conversely, a falling index value signals deteriorating market sentiment (i.e., falling risk appetite). While risk appetite is influenced by several factors, we believe that changing growth expectations are one of the primary drivers in global market sentiment. In fact, there is a strong positive correlation between our sentiment indicator and several proxy measures of the business cycle such as industrial production (.70 correlation), earnings per share momentum (.60) and our global leading economic indicator (.74), with lead times of 2-4 months and strong statistical significance (**Figure 50**).

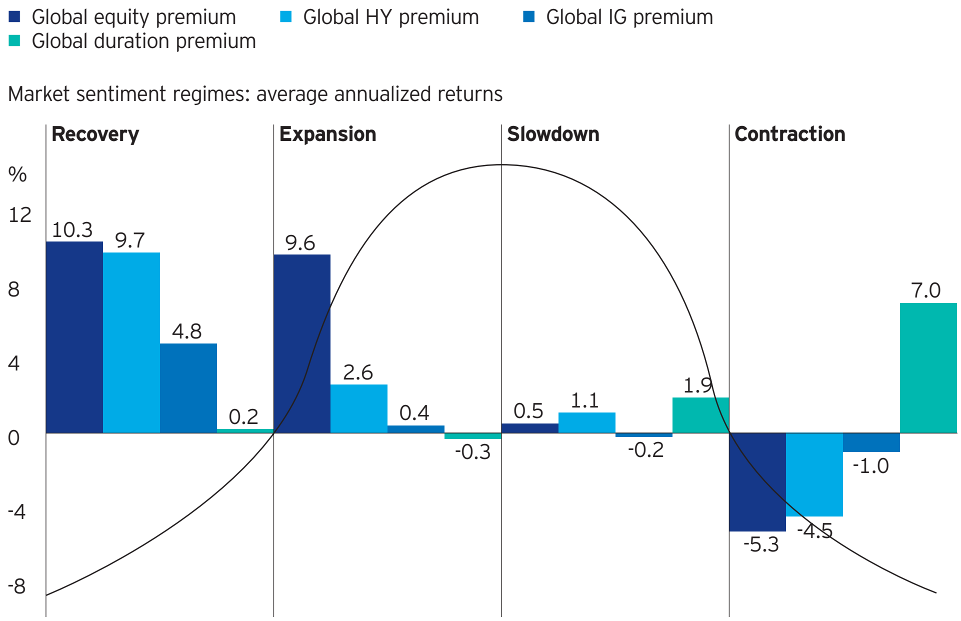
Figure 50: Strong correlation between investor confidence and the global business cycle
Global historical returns (Jan. 31, 1973–June 30, 2019)



Sources: Bloomberg L.P., MSCI, Citi, Barclays, JPMorgan, Invesco research and calculations, June 30, 2019. EPS momentum is calculated by comparing the relative magnitude of average EPS increases and decreases over the past six months. **Past performance does not guarantee future results.**

This strong parallel between global market sentiment and the global business cycle is confirmed by a historical analysis of the performance of traditional asset classes across macro regimes, defined using our market sentiment indicator in a similar fashion to our global leading economic indicator (**Figure 51**). These results confirm that global risk appetite can be used in conjunction with leading economic indicators to guide tactical asset allocation decisions, exploiting the leading nature of the market sentiment indicator to anticipate turning points in the growth cycle.

Figure 51: IIS Global Market Sentiment Regimes: Investment Implications
Global historical returns (Jan. 31, 1970–June 30, 2019)



Sources: Bloomberg L.P., Invesco, June 30, 2019. Global Equity Premium = MSCI ACWI Total Return - Global Treasuries 10Y Total Return, Global HY Premium = Global HY Total Return - Global Investment Grade Corporate Total Return, Global IG Premium = Global Investment Grade Corporate Total Return - Global Treasuries Total Return, Global Duration Premium = Global Treasuries 10Y Total Return - Global 3M T-bills. See Exhibit 5 for full methodology and disclosures for index definitions. Indices are unmanaged and cannot be purchased directly by investors. **Past performance does not guarantee future results.**

To assist investors with the difficult task of monitoring the economy and analyzing market movements, we propose a consistent tactical asset allocation framework, using leading economic indicators and market sentiment to anticipate turning points in economic growth, and reposition portfolio exposures across asset classes and risk premia consistent with the changing macro environment. Using this tactical framework, we aim to provide signal amid market noise and help make informed decisions within a short-to-intermediate timeframe.

08

Volatility and Correlation

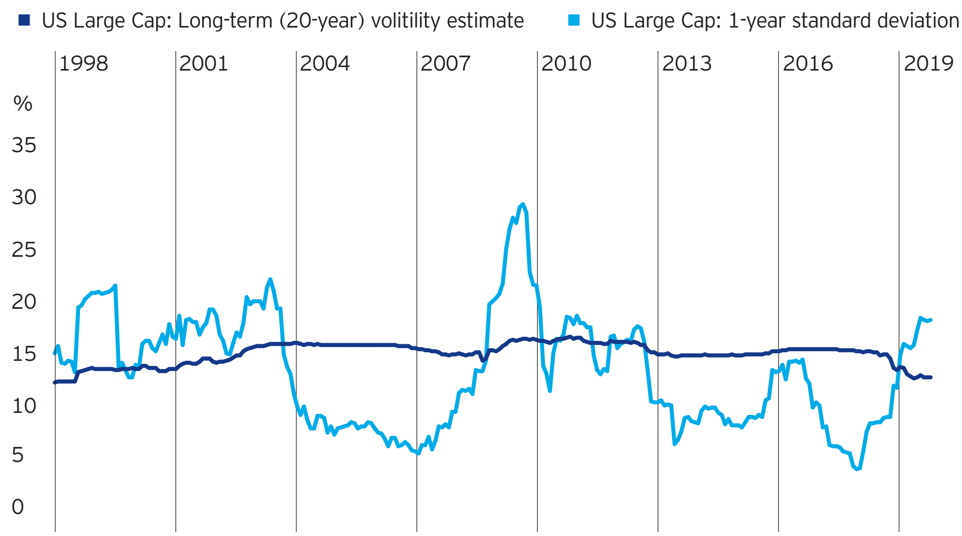
Volatility is estimated using rolling historical quarterly returns that are normalized for shorter lived benchmarks.

In order to construct multi-asset, goal-oriented portfolios that seek diversification and focus on specific investment outcomes, we also need to estimate the risk (i.e., volatility) of each asset class, as well as correlations between the different asset classes – how they move relative to each other. One commonly used methodology is to estimate risk and correlation directly from historical data.

Volatility

To estimate volatility for the different asset classes, we use rolling historical quarterly returns of various market benchmarks.

Figure 52: Mean-reverting properties of short-term volatility compared to long-term estimate



Source: Invesco, Sept. 30, 2019. **Past performance does not guarantee future results.**

Figure 53: Volatility is normalized for shorter lived benchmarks

$$\textit{US small cap volatility}^2 = \frac{\textit{Russell 2000 Index volatility}^1}{\textit{S\&P 500 Index volatility}^1} \times \textit{S\&P 500 Index volatility}^2$$

1 Sample periods are overlapping periods of S&P 500 and US Small Cap.
2 Sample periods are 1970-2017

Since all of these benchmarks have differing histories within and across asset classes, we normalized the volatility estimates of the shorter-lived benchmarks to ensure that all series are measured over similar periods. We did this by designating one benchmark to represent the full history for an asset class (**Figure 54**). The sub-asset classes with shorter histories are then adjusted based on their relationship to the representative benchmark. For example, to estimate the volatility of US smallcap equities over the entire history of the asset class dating back to 1970, we look at the relationship between the Russell 2000 Index (the benchmark for US small-cap equity) and the S&P 500 Index, as the representative benchmark for US equity, during the period in which they overlapped.

Figure 54: Benchmarks designated to represent the full history for an asset class

Asset class	Representative Index	History
US equity	S&P 500 Index	1970
International equity	MSCI EAFE Index	1970
US government bonds	BBG BARC US Treasury Index	1976
Corporate and other bonds	BBG BARC US Aggregate Index	1976
Commodities	S&P GSCI Index	1970

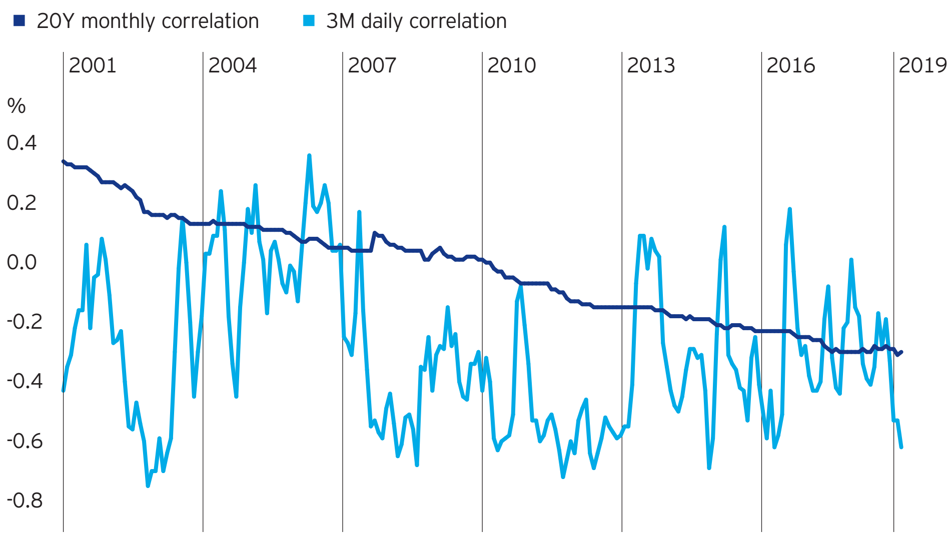
Full history dates shown include back-tested performance, which is hypothetical and subject to inherent limitations. The inception dates of the S&P 500 index, MSCI EAFE, BBG BARC US Treasury Index, BBG BARC US Aggregate Index, and S&P GSCI Index, respectively are; March 31, 1957, March 31, 1986, Jan. 31, 1973, Jan. 31, 1973, and Jan. 31, 1991.

Correlation

Correlation, or the extent to which asset classes move in the same direction, plays an important role in constructing a multi-asset portfolio that seeks to maximize the potential benefits of diversification. For our strategic capital market assumptions, we calculate correlation coefficients using the trailing 20 years of monthly index returns, which we believe is appropriate in covering a majority of asset classes while incorporating multiple business cycles.

A correlation coefficient is a statistical measure that can range in value from -1.0 (perfect negative correlation) to 1.0 (perfect positive correlation). It's important to recognize that correlations among asset classes can change over time. Since we believe that recent asset class correlations could have a more meaningful effect on future observations, we place greater weight on more recent observations by applying a 10-year half-life to the time series in our calculation.

Figure 55: Correlation trend persistance example for US Large Caps and US Treasuries



Invesco, as of Sept. 30, 2019. US Large Caps represented by the S&P 500 Index and US Treasuries represented by the Bloomberg Barclays Aggregate US Treasury Index.

09

Currency adjustments, expected returns and compound returns

Currency adjusted expected returns

Portfolios of an international or global nature will likely invest in financial instruments that are based in foreign currencies. For instance, a UK-based multi-asset portfolio manager will likely have an appreciable allocation to US large-cap equities based in USD. Since the UK-based manager wishes to consider his/her portfolio returns in terms of the local GBP currency, there is need to convert the forecasted returns for the US large-cap equity asset class from a USD-based perspective to a GBP-based perspective, especially for the purposes of optimal portfolio construction via mean-variance optimization or its robust counterpart.

For the UK-based portfolio manager, given an annualized expected return of μ_{USD} for the USD-based large-cap equities, and an annualized US government bond yield of i_{USD} and a similar annualized UK government bond yield i_{GBP} of our formulation for the annualized expected return in GBP is:

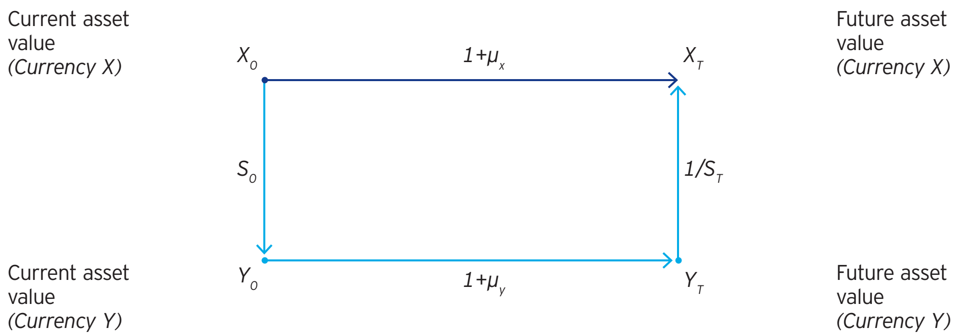
$$\mu_{GBP} = \mu_{USD} - i_{USD} + i_{GBP}$$

In what follows below, we provide the rationale for this return conversion.

At the core of our currency-based expected return conversion process is the concept of Interest Rate Parity. We utilize the basic concept that the future value of an asset denominated in currency *X* is equivalent to the foreign exchange rate-converted future value of the asset denominated in currency *Y*. **Figure 56** below graphically depicts such an equivalence.

Specifically, let X_O denote the current value of an asset denominated in currency *X* and let X_T denote its future value. Then, assuming a single period return of the future value is simply $X_T = (1 + \mu_X) X_O$. (This is the top dark blue segment in **Figure 56**.)

Figure 56: Interest rate parity commutation diagram



Source: Invesco, BarraOne.

An alternative to going directly from the current value X_0 to the future value X_T (in terms its return μ_X in currency *X*) is to first convert the value of X_0 in currency X to the value Y_0 in currency *Y*. Such a conversion may be simply expressed as $Y_0 = S_0 X_0$, where S_0 is the current foreign exchange rate in going from currency *X* to currency *Y*. (This is the left-most segment of Figure 28.) Next, assuming a single period return of μ_Y, the future value in currency *Y* is simply $Y_T = (1 + \mu_Y) Y_O$. (This is the bottom segment of **Figure 56**.) Finally, the future value Y_T may be converted to the future value X_T through a similar foreign exchange rate conversion. Namely, $X_T = Y_T/S_T$ where $1/S_T$ is the future foreign exchange rate going from currency *Y* to currency *X*. (This is the right-most segment of **Figure 56**.)

Since the future value of the asset denominated in currency *X* should be the same as the foreign exchange rate-converted future value of the asset denominated in currency *Y*, so as to not violate arbitrage conditions, this means:

$$x_T = x_O (1 + \mu_x) = S_O x_O (1 = \mu_y) (1/S_T)$$

If we perform the same analysis along the same paths, now in terms of two government bonds (whose returns we treat as certain), one denominated in currency *X* with yield i_x and the other in currency *Y* with yield i_y, then we will have:

$$\frac{1 + \mu_X}{1 = \mu_Y} = S_O S_T = \frac{1 + i_X}{1 + i_Y}$$

Nothing that $(1 + \mu_X) (1 + \mu_Y)^{-1} \approx 1 + \mu_X - \mu_Y$, *and similarly that* $(1 + i_X) (1 + i_Y)^{-1} \approx 1 + i_X \, 1 + i_Y$, *means*

$$\mu_Y = \mu_X - i_X + i_Y$$

Since our portfolio construction perspective is a strategic, long-horizon one, we use the annualized yields of the 10-year government bonds in currencies *X* and *Y* in the above return conversion formula and combine them with the annualized forecasted return in currency *X*. This is our estimate of the forecasted annualized return in currency *Y*. This modeling assumption leads to similar return estimates, whether we choose to hedge or not. Of course, from a risk perspective, currency hedging will have a meaningful impact.

Arithmetic versus geometric returns

In practice, asset returns are most commonly expressed in geometric terms. This is because the investors are most often concerned with either the rate at which an investment grew in the past or the rate it might be expected to grow in the future (or over the long term). The geometric mean return is the average rate of return per period when returns are compounded over multiple periods. Consider a time series of returns r_t, *for* $t = 1, 2, \dots, T$ periods, and some initial investment amount W_0. The value of the investment at time T *is* $W_T = W_O \times (1 + r_1) \times (1 + r_2) \dots \times (1 + r_T)$. The geometric return μ_g, or geometric mean, of such a time series is then:

$$\mu_g = \left(\prod_{t=1}^{T} (1 + r_t) \right)^{1/T} - 1$$

The geometric mean return is of interest to investors because it neatly expresses the periodic growth rate of a time series, (i.e., $W_T = W_O (1+\mu_g)^T$. This is of practical importance in terms of understanding the desirability of one investment over another. However, the geometric mean says nothing about risk, or rather, the variability of the returns an investor might actually receive from one period to the next. In fact, two assets can have the same geometric mean but exhibit substantially different variability of returns. To consider risk we must understand the expected value of the return we might receive in any period along with the variability around that expected value. This is where expressing returns in arithmetic terms is useful for investors.

The arithmetic mean μ_g is just the simple average of the periodic returns produced by an asset over a specified investment horizon and is calculated as:

$$\mu_a = \frac{1}{T} \sum_{t=1}^{T} r_t$$

This is particularly important for portfolio construction as it describes the probability-weighted return outcome (central tendency) of a return distribution, or rather, its expected return. If the returns provided by a particular return distribution were all equally likely, then the geometric mean could serve as our expectation. However, returns for most risky financial assets are not equally likely as they exhibit some degree of variability. This variability is most commonly expressed as a function of standard deviation. It can be shown that $\mu_a > \mu_g$ when the standard deviation of a return series is greater than zero. This highlights the fact that the volatility of a return series provides a link between the arithmetic return and the geometric return. Markowitz and Blay (2013) explore various mean-variance approximations to the geometric mean and find that the following approximation provides a reasonable generalization of this relationship:

$$\mu_g = e^{\ln(1+\mu_a) - \frac{\frac{1}{2}\sigma^2}{(1+\mu_a)^2}} - 1 \approx \mu_a - \frac{1}{2}\sigma^2$$

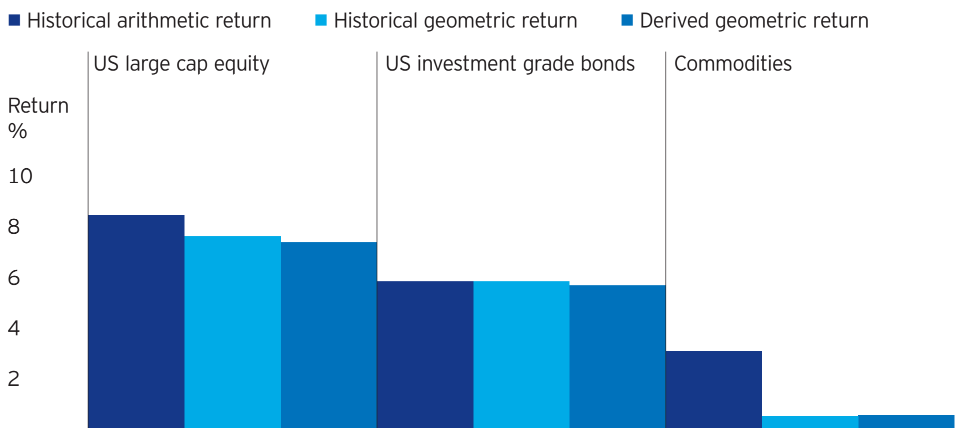
This approximation allows investors to go back and forth between arithmetic and geometric returns as long as they know an asset's or portfolio's arithmetic mean μ_a and volatility σ. It should be noted that using the historical information (e.g., arithmetic means, standard deviations, and correlations) in a portfolio analysis will produce portfolios that will have likely performed well in the past. Expected returns should represent expectations for returns that are likely to be achieved in the future expressed in arithmetic terms. The approximation above can also be helpful in producing expected return estimates that are appropriate for use in a portfolio analysis as well as being aligned with intuition in geometric terms.

As an example of how well such a simple approximation can work, in **Figure 57**, we consider the historical arithmetic and geometric returns for three standard asset classes:

1. US Large-Cap Equity
2. US Investment-Grade Bonds and,
3. Commodities and compare the historical geometric return with one derived from the approximation above.

The two geometric returns are very close and differ by no more than 10.5 basis points in this example.

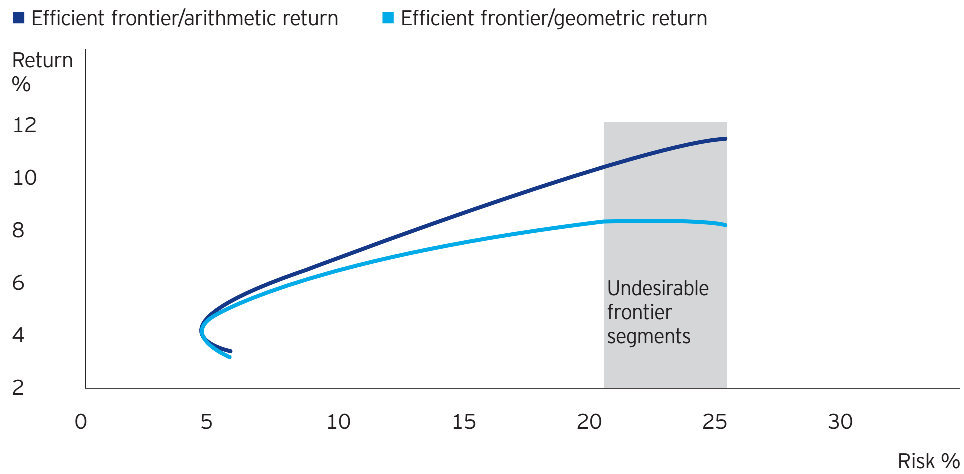
Figure 57: Historical arithmetic, geometric and derived geometric returns for select asset classes



Source: Invesco, Bloomberg L.P., Monthly return data period from Sept. 1, 1998 to Aug. 31, 2018. Note: The historical volatilities of the asset classes over the period are as follows: US Large Cap Equity 14.5%, US Investment Grade Bonds 3.5% and Commodities 22.5%. US Large Cap is represented by the S&P 500 Index, US Investment Grade Bonds is represented by the BBG BARC Aggregate Bond Index, and Commodities are represented by the BBG Commodities Index. **Past performance does not guarantee future results.**

The ability to effectively translate arithmetic returns to geometric returns (and vice versa) is of consequence to investors as the return inputs, or expected returns, used in a mean-variance portfolio optimization must necessarily be expressed in arithmetic terms. The reason for this is that the arithmetic means of a weighted sum (e.g., a portfolio) is the weighted sum of the arithmetic means (of the portfolio constituents). This does not hold for geometric returns. In other words, the weighted average of the arithmetic means of the assets included in a portfolio is equal to the arithmetic mean of the portfolio as a whole. This is not the case when geometric means are used. Since the expected return inputs of a portfolio analysis are required to be in arithmetic terms, the outputs of such an analysis are also in arithmetic terms and must be translated, through the use of the portfolio mean and standard deviation, into the more intuitive geometric terms that describe the expected growth rates provided by the efficient set of portfolios for portfolio selection. **Figure 58** presents an example of an efficient frontier presented in both arithmetic and geometric terms.

Figure 58: Efficient frontier presented in arithmetic and geometric terms



Source: Invesco. For illustrate purposes only.

Note that the efficient frontier expressed in terms of arithmetic returns sits well above the efficient frontier expressed in terms of geometric returns. This is so because the geometric returns are downward adjustments of the arithmetic returns. It is only when we view the efficient frontier expressed in this fashion that we can see how, at segments of the frontier where portfolio volatility is sufficiently large, pursuing portfolios with higher arithmetic returns can result in the likelihood of achieving lower long-term (geometric) returns than portfolios with lower risk.

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